

2017-18 Onwards (MR-17)	MALLA REDDY ENGINEERING COLLEGE (Autonomous)	B.Tech. III Semester		
Code: 70308	KINEMATICS OF MACHINES	L	T	P
Credits: 3		2	2	-

Prerequisites: Engineering Mechanics & Engineering Graphics

Course Objectives:

The objective of this subject is to provide basic concept of Mechanisms used in different machine elements like cams, gears and other power transmitting elements.

MODULE I: Mechanisms & Machines 10 Periods

Mechanisms: Elements or Links, Classification, Rigid Link, flexible and fluid link, Types of kinematic pairs, sliding, turning, rolling, screw and spherical pairs, lower and higher pairs, closed and open pairs, constrained motion, completely, partially or successfully constrained and incompletely constrained.

Machines: Mechanism and machines, classification of machines, kinematic chain, inversion of mechanisms, inversions of quadratic cycle chain, single and double slider crank chains.

MODULE II: Straight Line Motion Mechanisms, Steering Mechanisms & Hooke's Joint 10 Periods

Straight Line Motion Mechanisms: Exact and approximate copiers and generated types Peaucellier, Hart and Scott Russel, Grasshopper, Watt T.Chebicheff and Robert Mechanisms and straight line motion, Pantograph.

Steering Mechanisms: Conditions for correct steering, Davis Steering gear, Ackerman's steering gear mechanisms, velocity ratio. Hooke's Joint – Single and double Hooke's joint, Universal coupling application problems.

MODULE III: Kinematics & Plane motion of body 10 Periods

A: Kinematics: Velocity and acceleration - Motion of link in machine - Determination of Velocity and acceleration diagrams - Graphical method - Application of relative velocity method four bar chain. Analysis of Mechanisms: Analysis of slider crank chain for displacement, velocity and acceleration of slider - Acceleration diagram for a given mechanism, Kleins construction, Coriolis acceleration, determination of Coriolis component of acceleration.

B: Plane motion of body: Instantaneous centre of rotation, centroids and axodes - relative motion between two bodies - Three centres in line theorem - Graphical determination of instantaneous centre, diagrams for simple mechanisms and determination of angular velocity of points and links.

MODULE IV: Cams & Analysis of Motion of Followers 09 Periods

Cams: Definitions of cam and followers, their uses, types of followers and cams, terminology. Types of follower motion - Uniform velocity, Simple harmonic motion and uniform acceleration. Maximum velocity and maximum acceleration during outward and return strokes in the above three cases.

Analysis of motion of followers: Roller follower circular cam with straight, concave and convex flanks.

MODULE V: Gears & Gear Trains

09 Periods

Gears: Higher pairs, friction wheels and toothed gears, types, law of gearing, condition for constant velocity ratio for transmission of motion, Forms of teeth: cycloidal and involute profiles. Velocity of sliding phenomena of interferences, Methods of interference. Condition for minimum number of teeth to avoid interference, expressions for arc of contact and path of contact - Introduction to Helical, Bevel and worm gearing.

Gear Trains: Introduction - Train value - Types - Simple and reverted wheel train, Epicyclic gear train. Methods of finding train value or velocity ratio - Epicyclic gear trains. Selection of gear box - Differential gear for automobiles.

TEXT BOOKS

1. Thomas Bevan “**Theory of Machines**”, CBS Publishers, 3rd edition, 2005
2. S.S.Rattan “**Theory of Machines and Mechanisms**”, Tata McGraw Hill Publishers, 4th edition, 2014.

REFERENCES

1. Shiegley “**The Theory of Machines**”, Oxford University Press, 5th edition, 2017.
2. JS Rao and RV Dukkupati, “**Mechanism and Machine Theory**”, New Age International Publishers, 2nd edition, 1992.
3. R.K Bansal “**Theory of Machines**”, Laxmi Publication, 4th edition, 2006
4. R.S. Khurmi and J K Gupta “**Theory of Machines**”, S Chand Publisher, 14th edition, 2008.
5. B.V. R. Gupta, “**Theory of Machines**”, I. K. International Publishers, 2nd edition, 2011

E - RESOURCES

1. www.umt.fme.vutbr.cz/~ruja/vyuka/kinematics/LectureNotes.pdf.
2. ebooks.library.cornell.edu/k/kmoddl/pdf/016_002.pdf
3. www.springer.com/la/book/9789400711556
4. tps://www.elsevier.com/journals/mechanism-and-machine-theory/0094-114X?...
5. www.nptelvideos.in/2012/12/kinematics-of-machines.html
6. nptel.ac.in/courses/112104121/16

Course Outcomes

At the end of the course, students will be able to

1. Understand different kinematic links and pairs used in mechanisms.
2. Understand different mechanisms.
3. Understand various concepts of mechanisms like straight line motion mechanisms, steering gear mechanisms and the importance of universal (Hooke's) joint.
4. Practice velocity, acceleration diagram of various mechanism.
5. Understand various motions of cam and follower and belt and rope drives.

KINEMATICS OF MACHINES
LECTURE NOTES
UNIT – I&II

Link or Element :

A link is a resistant body or assembly of resistant bodies, which constitute part or parts of a machine , connecting other parts which have motion , relative to it.

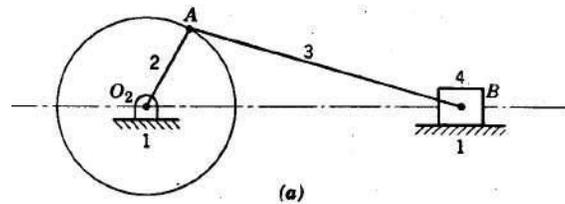
A link may consist of a number of parts connected in such a way that they form one unit and have no motion relative to each other

eg. Piston , piston rod and cross head of a steam engine constitute one unit and will be considered as one link. Similarly the crank pin, crank webs, crankshaft, flywheel etc constitute another link known as a crank. On the other hand crosshead, connecting rod, crank and frame of a steam engine are other links.

In fig.

Link 1 -> fixed link link 2 -> crank

link 3 -> connecting rod link 4 -> slider



Single slider crank mechanism

Types of links :

Classification I :

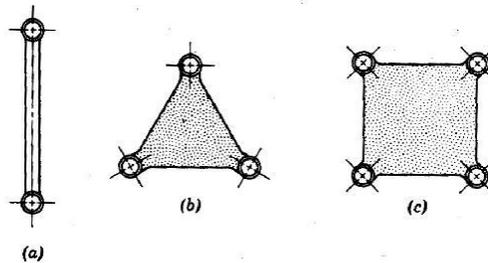
Rigid link is one which does not undergo any deformation while transmitting motion. In the real sense, rigid links do not exist. However, as the deformation of connecting rod, crank etc of a steam or internal combustion engine is not appreciable they can be considered as rigid.

Flexible link is one which while transmitting motion is partly deformed in a manner not to affect transmission of motion. Thus it transmits motion in one direction only so as to pull or push e.g. belt.

Fluid link is formed by having fluid in a receptacle and the motion is transmitted through the fluid by pressure or by compression only as in a hydraulic press.

Classification II :

. **Binary link:** Link which is connected to other links at two points. (Fig.a)



Ternary link: Link which is connected to other links at three points. (Fig. b)

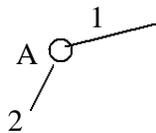
Quaternary link: Link which is connected to other links at four points. (Fig c)

Joint and types of Joint:

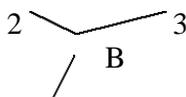
A point which connects two links is called joint

There are three types of joints .

Binary Joint : If any joint connects two links then it is called binary joint. In fig. joint A is connected to two links 1 and 2

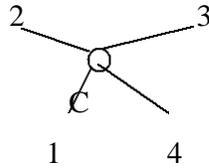


Ternary Joint : If any joint connects three links then it is called ternary joint. In fig. Joint B is connected to links 1,2&3

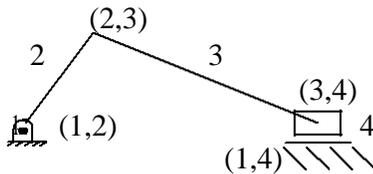


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Quaternary Joint : If any joint connects four links then it is called quaternary joint. In fig. joint C is connected to links 1,2,3 &4



Kinematic pair : It is a joint of two links having relative motion between them.



In a slider crank mechanism as shown , fixed link 1 and crank 2 form one pair , crank2 and connecting rod 3 form another pair , connecting rod 3 and slider 4 form another pair and finally

Classification of Kinematic pairs :

A kinematic pair is classified according to the following considerations

a) Type of contact b) type of relative motion c) type of mechanical constraint.

a) **According to Type of Contact** : According to this there are two types of pairs.

1. **Lower pairs** : A pair having surface or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

E.g. Nut turning on a screw, shaft rotating in a bearing ,all pairs of slider crank mechanism, universal joint etc

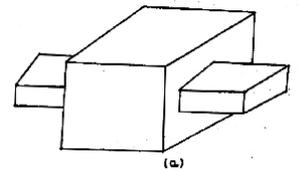
2. **Higher Pairs**: When a pair has a point or line contact between the links , it is known as higher pair. The contact surfaces of the two links are dissimilar.

E.g. Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings etc

b) **According to Type of relative motion** : according to this ,there are five types of pairs.

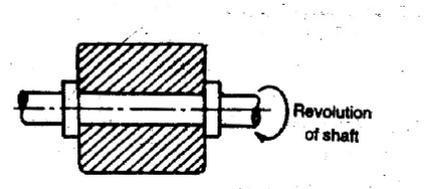
1. **Sliding Pair** : If two links have a sliding motion relative to each other , they form a sliding pair

E.g. A rectangular rod in a rectangular hole in a prism



2. **Turning pair** : When one link has a turning or revolving motion relative to the other , they constitute a turning or revolving pair.

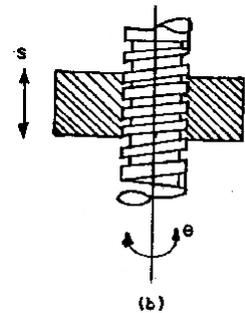
E.g. A circular shaft revolving inside a bearing is a turning pair, All pairs except the slider and guide pair are turning pairs in a single slider crank mechanism



3. **Rolling pair** : When the links of a pair have a rolling motion relative to each other, they form a rolling pair, e.g. A rolling pulley over a belt, a ball and rolling bearings etc . In a ball bearing , the ball and the shaft constitute one rolling pair where as the a ball and the bearing is the second rolling pair.

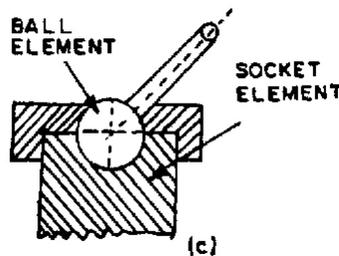


4. **Screw pair or helical pair** : Iff two mating links have a turning as well as sliding motion between them, they form a screw pair. This is achieved by cutting matching threads on the two link. The lead screw and the nut of a lathe, a bolt and nut are screw pairs.



5. **Spherical Pair** : When one link in the form of a sphere turns inside a fixed linkk, it is a spherical

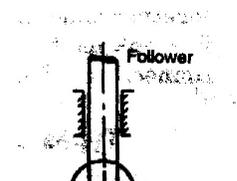
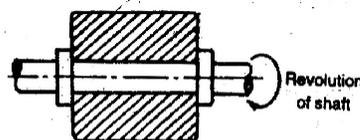
E.g. The a ball and socket joint



c)According to Nature of mechanical constraint : there are two types

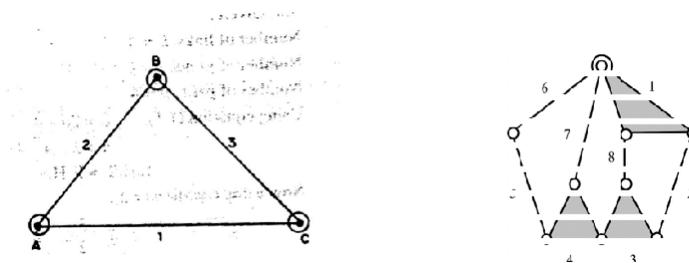
1.**Closed pair** : when the elements of a pair are held together mechanically , it is known as a closed pair. The contact between the two can be broken only by destruction of at least one of the members.

All the lower pairs and some of the higher pairs are closed pairs.



2. **Unclosed pair(Open Pair)** : Whhen two links of a pair are in contact either due to force of gravity or some spring action they constitute an unclosed pair. In this , the links are not held together mechhnically eg. Cam and follower pair

Kinematic Chain: A kinematic chhain is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the others is definite.



If the links are connected in such a way that no motion is possible, it results in a locked chain or structure.

If each link is assumed to form two pairs with two adjacent links, then the relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation :

$$l = 2p - 4$$

If each link is assumed to form two pairs with two adjacent links, then the relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation :

$$j = \frac{3}{2} l - 2$$

The equations (i) and (ii) are applicable only to kinematic chains, in which lower pairs are used. These equations may also be applied to kinematic chains, in which higher pairs are used. In that case each higher pair may be taken as equivalent to two lower pairs with an additional element or link.

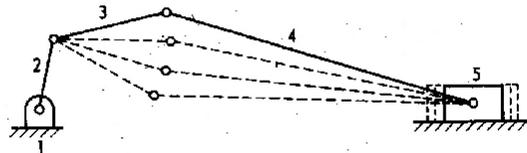
Mechanism or Linkage :

When any one link is fixed , the kinematic chain becomes a mechanism



Slider crank and four bar mechanisms

If, for a particular position of a link of the chain, the positions of each of the other links of the chain can not be predicted, then it is called as unconstrained kinematic chain and it is not mechanism.



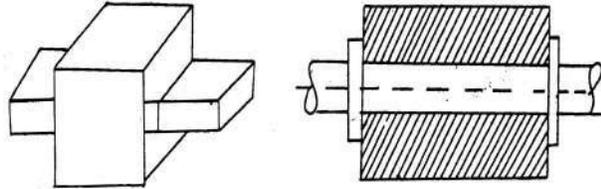
Difference between mechanism and machine

Mechanism	Machine
<p>1. A mechanism is an assemblage of links, the motion to one of them producing definite motions of others.</p> <p>2. A mechanism is not necessarily a machine. It is meant to produce definite motion between various links constituting it.</p> <p>3. A Mechanism is a part of a machine</p> <p>e.g. a watch , wiper mechanism</p>	<p>1. A machine is an assemblage of links having relative motion and capable of transforming available energy in to certain useful work.</p> <p>2. A machine is constituted of a mechanism or a number of mechanisms meant for transmitting energy to do useful work</p> <p>3. A machine is a practical development of any mechanism</p> <p>e.g. steam engine</p>

There are three types of constrained motions

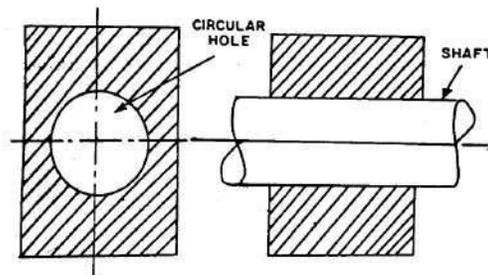
a. Completely constrained motion: It is one in which the motion takes place in a definite direction.

e.g. a rectangular bar moving in a rectangular hole. And a shaft with collars at each end rotating in a round hole.

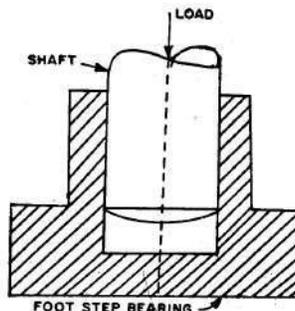


b. Incompletely constrained motion : If the links are so connected that motion can take place in more than one direction, it is called incompletely constrained motion. A

Eg. A circular bar moving in a round hole because the bar can rotate or reciprocate , both motions being independent of one another.



c. Partially constrained motion (Successfully Constrained motion): When the constrained motion is not completed by itself but by some other means (ie by applying some external force), it is partially constrained or successfully constrained motion. Eg. A foot step bearing and rotor of a vertical turbine.



Grublers Criterion:

Let n = total number of links in a mechanism

F is degrees of freedom

P_1 is the no. of pairs having one degree of freedom

P_2 is the no. of pairs having two degrees of freedom

Total number of movable links = $n-1$

No of degrees of freedom of $(n-1)$ movable links is $3(n-1)$

Each pair having one dof impose 2 restrictions .

So no of restrictions because of P_1 pairs are $2P_1$

Each pair having two dof impose 1 restrictions .

So no of restrictions because of P_2 pairs are $1P_2$

Hence ,

total degrees of freedom for mechanism = $3(n-1) - 2P_1 -$

$1P_2$ This equation is known as **Grueblers criterion**.

If all the pairs are of one dof , thhen $P_2=0$

then dof = $3(n-1) - 2P_1$

Dof = 0 means there is no relative motion between the links and it is called locked chain or structure

Dof = 1 means one input is necessary for definite motion of other links.

Dof = 2 means two inputs are necessary for definite motion of other links.

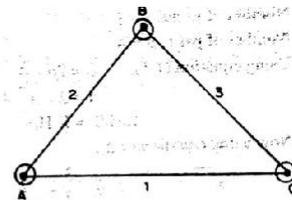
Dof = -1 then the mechanism is called super structure.

eg 1.

$$\text{dof} = 3(n-1) - 2P_1$$

Here, $n = 3, P_1 = 3$

$$F = 3(3-1) - 2(3) = 0$$

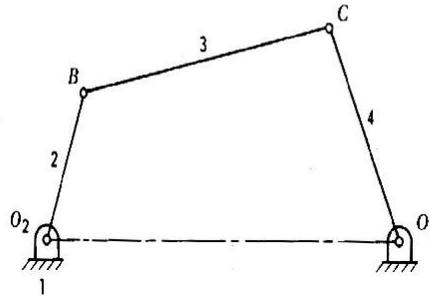


Eg 2.

$$\text{dof} = 3(n-1) - 2P_1$$

Here, $n = 4, P_1 = 4$

$$F = 3(4-1) - 2(4) = 1$$

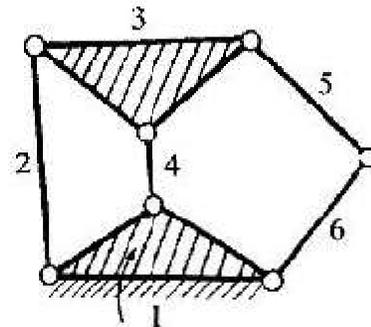


$$F = 3(n-1) - 2l - h$$

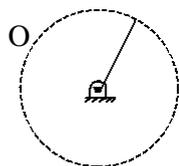
Here, $n_2 = 4, n_3 = 2, n = 6, l = 7$ and $h = 0$.

$$F = 3(6-1) - 2(7) = 1$$

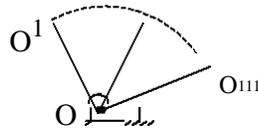
I.e., one input to any one link will result in definite motion of all the links.



Crank : Any link which can rotate about hinge point completely is called crank . In fig. link 1 rotates completely about hinge point O so it is crank



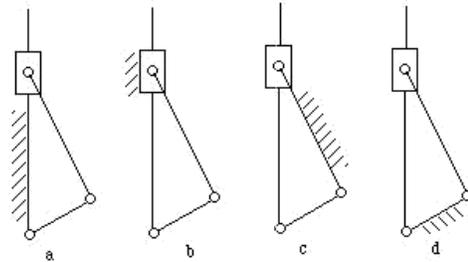
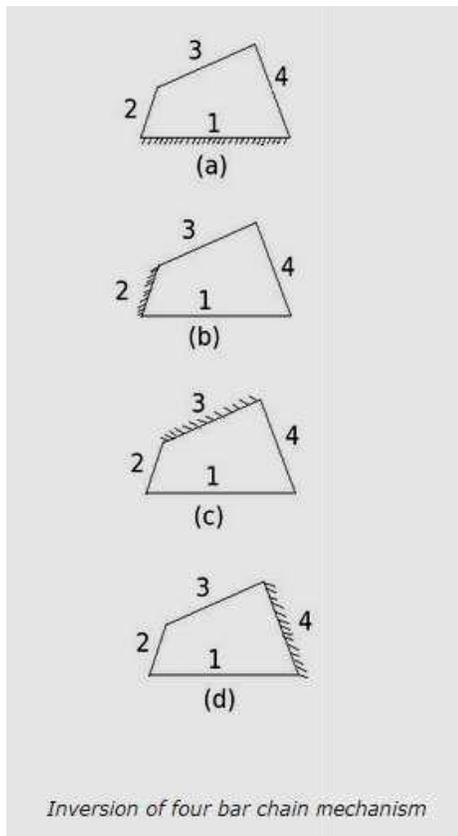
Lever or Rocker : Any link which oscillates between two extreme positions is called a lever or rocker . In fig. link 1 can only oscillate between $O O^I$ and $O O^{III}$ so it is rocker or lever



Inversion

A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism.

By changing the fixed link, the number of mechanisms which can be obtained is equal to the number of links. Excepting the original mechanism, all other mechanisms will be known as inversions of original mechanism. The inversion of a mechanism does not change the motion of its links relative to each other.



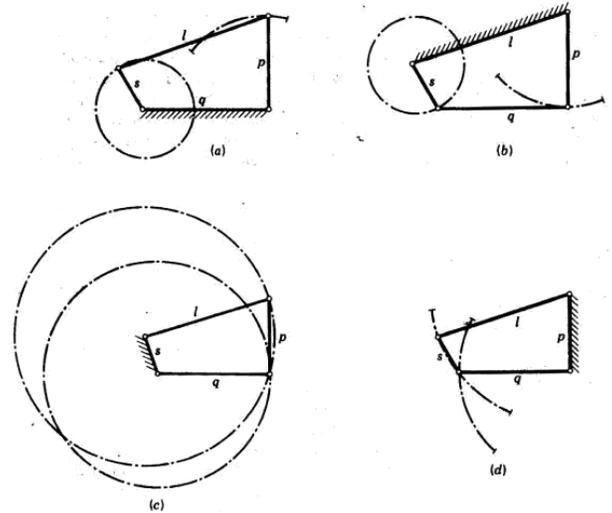
Inversions four bar kinematic chain (quadric cycle kinematic chain) :

There are four inversions of four bar chain

i) when link adjacent to the shortest link is fixed (ie link 'l' or 'q') , then the shortest link (ie link 's') act as crank and the link parallel to the shortest link (ie link 'p') act as lever or rocker . This mechanism is called crank rocker mechanism (**fig a and fig b**)

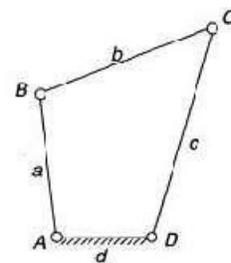
ii) when shortest link is fixed (ie link 's'), then the links adjacent to the shortest link (ie link 'l' and link 'q') act as cranks. This mechanism is called double crank mechanism (**fig c**)

iii) when link opposite to the shortest link is fixed (ie link 'p'), then the the links adjacent to the shortest link (ie link 'l' and link 'q') act as levers or rocker . This mechanism is called double rocker mechanism (**fig d**)



Grashof Law states that a four bar mechanism has atleast one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of lengths of the other two links.

For four bar mechanism as shown
 $c+d < a+b$

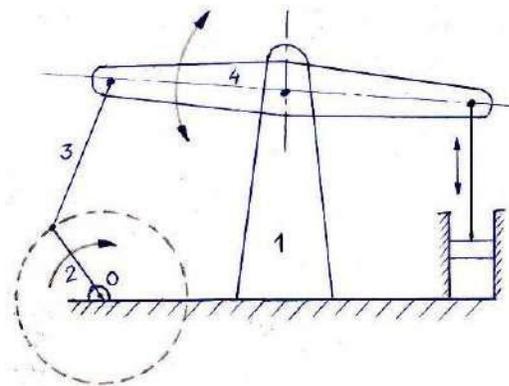


Applications :

1. Beam Engine

A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig..

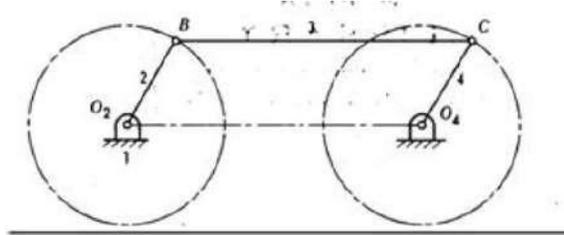
In this mechanism, when the crank 2 rotates about the fixed centre O, the lever 4 oscillates about a fixed centre. The end of the lever is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion



2. Parallel Crank Mechanism :

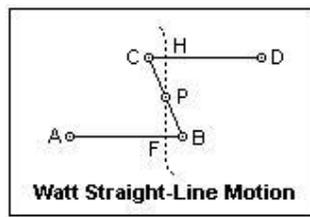
In this mechanism, the links O_2B and O_4C (having equal length) act as cranks and are connected to the respective wheels.

The link BC acts as a coupling rod and the link O_2O_4 is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.



3. Watt straightline motion mechanism:

It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion. When links AB and CD oscillate, then the point P traces an approximate straight line.



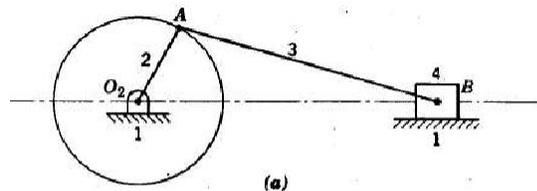
Inversions and applications of single slider crank kinematic chain

Slider crank chain: This is a kinematic chain having four links. It has one sliding pair and three turning pairs.

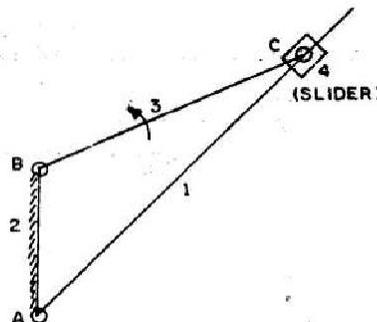
Link 2 has rotary motion and is called crank. Link 3 has got combined rotary and reciprocating motion and is called connecting rod. Link 4 has reciprocating motion and is called slider. This mechanism is used to convert rotary motion to reciprocating and vice versa.

Inversions of slider crank chain: Inversions of slider crank mechanism is obtained by fixing links 2, 3 and 4.

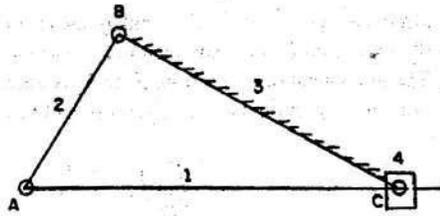
(a) Link 1 is fixed.



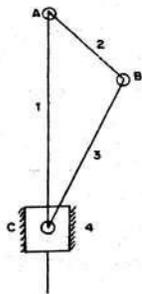
(b) crank fixed



(c) connecting rod fixed



(d) slider fixed

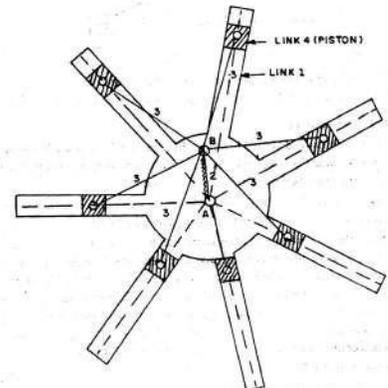


Applications :

1. Rotary engine (Gnome Engine)

It consists of seven cylinders in one plane and all revolves about fixed centre *D*, as shown in Fig., while the crank (link 2) is fixed.

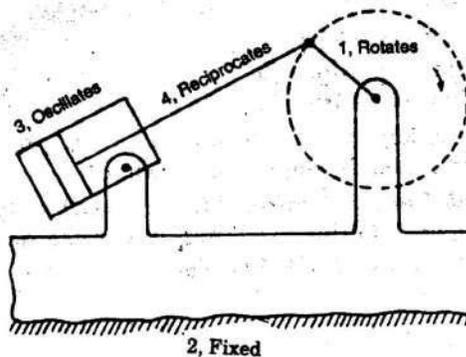
In this mechanism, when the connecting rod (link 3) rotates, the piston (link 4) reciprocates inside the cylinders forming link 1.



2. Oscillating cylinder engine:

The arrangement of oscillating cylinder engine mechanism, as shown in Fig. , is used to convert reciprocating motion into rotary motion.

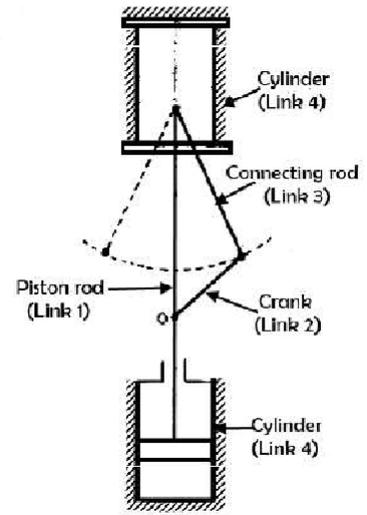
In this mechanism, the link 3 forming the turning pair is fixed. When the crank (link 1) rotates, the piston attached to piston rod (link 4) reciprocates and the cylinder (link 3) oscillates about a pin pivoted to the fixed link.



3. Hand Pump:

crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates.

The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig

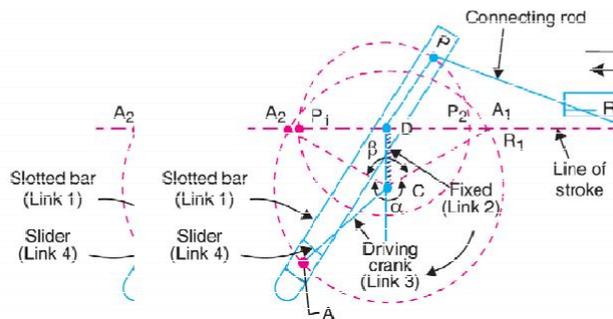


4. Whitworth quick return motion mechanism :

This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in Fig.

Time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha}$$

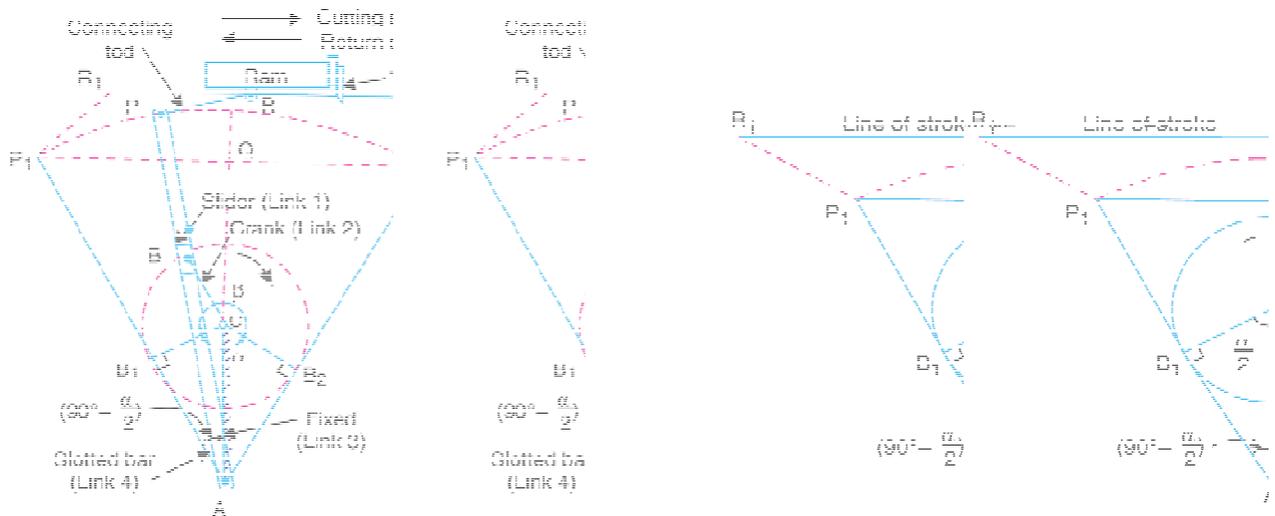


5. Crank and Slotted Lever Mechanism:

This mechanism is mostly used in shaping machines and slotting machines

In this mechanism, the link AC (*i.e.* link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C.

A sliding block attached to the crank pin at B slides along the slotted bar APP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (*i.e.* R1R2) is perpendicular to AC produced



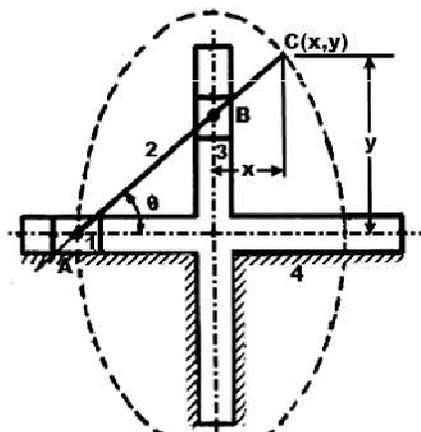
In the extreme positions, APP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta}$$

- AC= Length of fixed link
- AP_1 = Slotted Lever Length
- CB_1 = Crank Length
- $\tan (90 - \alpha/2) = QP_1/AP_1$
- $\cos (\alpha/2) = B_1C/AC$
- Stroke Length = $2(QP_1)$

Inversions and applications of double slider crank kinematic chain :

1. Elliptical trammel. This is a device which is used for generating an elliptical profile



When the sliders reciprocate, the point C traces elliptical path.

Let (x,y) be the coordinates of the point C

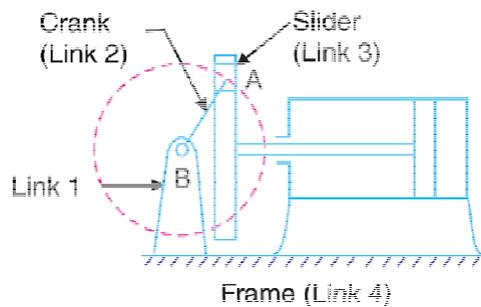
$$x = BC \cos \theta \Rightarrow \cos \theta = \frac{x}{BC}$$

$$y = AC \sin \theta \Rightarrow \sin \theta = \frac{y}{AC}$$

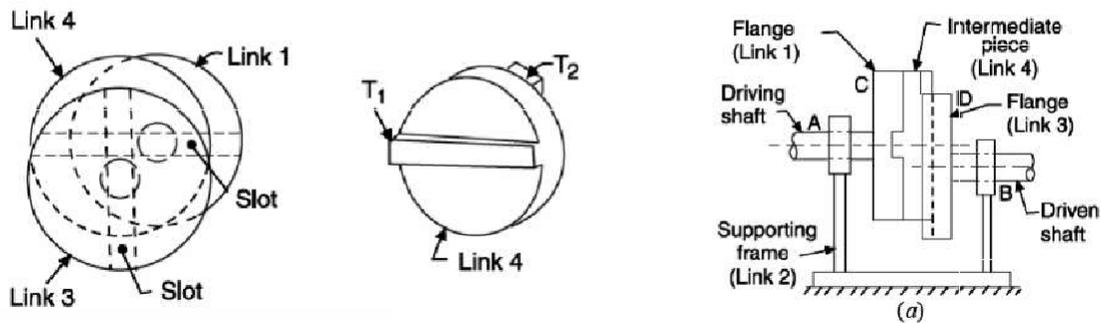
This is the equation for ellipse. Hence the point C traces elliptical path.

2.Scotch Yoke Mechanism:

This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig. link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.



3. Oldham Coupling:



An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Fig. .

If the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts.

Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

Let ω = Angular velocity of each shaft in rad/s, and r = Distance between the axes of the shafts in metres.

Maximum sliding speed of each tongue (in m/s), $v = \omega \cdot r$

Pantograph:

A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point. It consists of a jointed parallelogram $ABCD$ as shown in Fig. It is made up of bars connected by turning pairs. The bars BA and BC are extended to O and E respectively, such that $OA/OB = AD/BE$

Thus, for all relative positions of the bars, the triangles OAD and OBE are similar and the points O , D and E are in one straight line. It may be proved that point E traces out the same path as described by point D .

From similar triangles OAD and OBE , we find that

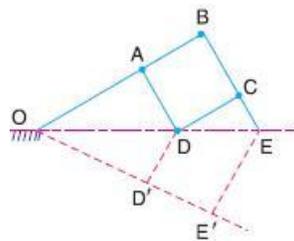
$$OD/OE = AD/BE$$

Let point O be fixed and the points D and E move to some new positions D' and E' . Then

$$OD'/OE' = OD/OE$$

A little consideration will show that the straight line DD' is parallel to the straight line EE' . Hence, if O is fixed to the frame of a machine by means of a turning pair and D is attached to a point in the machine which has rectilinear motion relative to the frame, then E will also trace out a straight line path. Similarly, if E is constrained to move in a straight line, then D will trace out a straight line Parallel to the former.

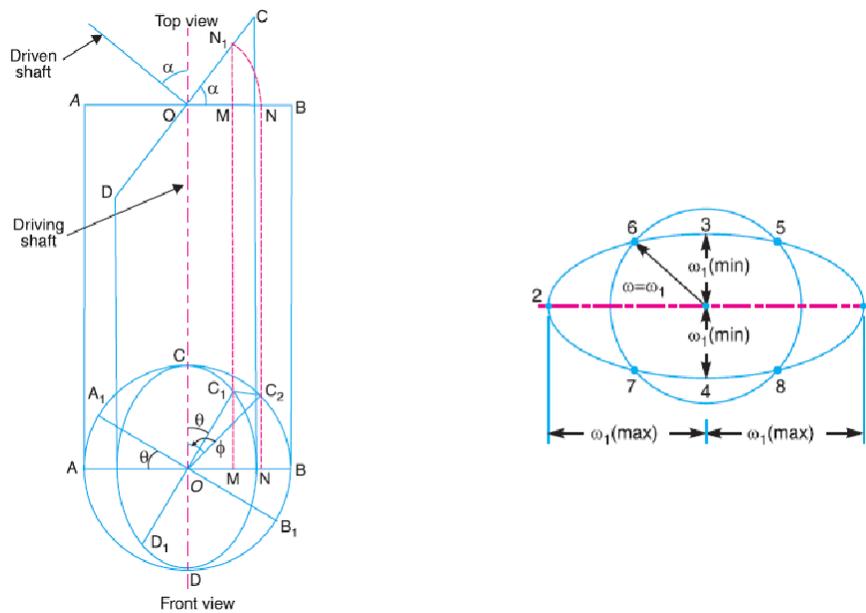
A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc., on enlarged or reduced scales. It is, sometimes, used as an indicator rig in order to reproduce to a small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine. It is also used to guide cutting tools. A modified form of pantograph is used to collect power at the top of an electric locomotive.



Pantograph

Universal or Hooke's Joint :

A Hooke's joint is used to connect two shafts, which are intersecting at a small angle, as shown in Fig The end of each shaft is forked to U-type and each fork provides two bearings



for the arms of a cross. The arms of the cross are perpendicular to each other. The motion is transmitted from the driving shaft to driven shaft through a cross. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted. The main application of the Universal or Hooke's joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machine. It is also used as a knee joint in milling machines.

In case of automobiles, we use two Hooke's joints one at each end of the propeller shaft, connecting the gear box on one end and the differential on the other end. The top and front views connecting the two shafts by a universal joint are shown in Fig.. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts. Let the driving shaft rotates through an angle θ , so that the arm AB moves in a circle to a new position A₁ B₁ as shown in front view. A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm CD takes new position C₁D₁ on the ellipse, at an angle θ . But the true angle must be on the circular path. To find the true angle, project the point C₁ horizontally to intersect the circle at C₂. Therefore the angle COC₂ (equal to ϕ) is the true angle turned by the driven shaft. Thus when the driving shaft turns through an angle θ , the driven shaft turns through an angle ϕ . It may be noted that it is not necessary that ϕ may be greater than θ or less than θ . At a particular point, it may be equal to θ .

The relation between angular displacements of driving and driven is given from above diagram

$$\tan \theta = \cos \alpha \tan \phi$$

Maximum and Minimum Speeds of Driven Shaft :

1. For one complete revolution of the driven shaft, there are two points *i.e.* at 0° and 180° as shown by points 1 and 2 in Fig. 9.20, where the speed of the driven shaft is maximum and there are two points *i.e.* at 90° and 270° as shown by point 3 and 4 where the speed of the driven shaft is minimum.

2. Since there are two maximum and two minimum speeds of the driven shaft, therefore there are four points when the speeds of the driven and driver shaft are same. This is shown by points, 5, 6, 7 and 8 in Fig.

3. Since the angular velocity of the driving shaft is usually constant, therefore it is presented by a circle of radius ω . The driven shaft has a variation in angular velocity, the maximum value being

$\omega/\cos \alpha$ and minimum value is $\omega \cos \alpha$. Thus it is represented by an ellipse of semi-major axis $\omega/\cos \alpha$ and semi-minor axis $\omega \cos \alpha$, as shown in Fig.

$$\frac{\omega_2}{\omega_1} = \frac{\cos \alpha}{1 - \sin^2 \alpha \cos^2 \theta}$$

The relation between driving and driven shaft angular velocities are

(i) if $\frac{\omega_2}{\omega_1} = 1$ then

$$\tan \theta = \sqrt{\cos \alpha}$$

(ii) if $\frac{\omega_2}{\omega_1}$ is minimum

$$\frac{\omega_2}{\omega_1} = \cos \alpha$$

(iii) if $\frac{\omega_2}{\omega_1}$ is maximum

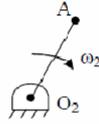
$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos \alpha}$$

For maximum angular acceleration of driven shaft, the condition is

$$\cos 2\theta = \frac{2 \sin^2 \alpha}{2 - 2 \sin^2 \alpha}$$

UNIT – III
VELOCITY ANALYSIS

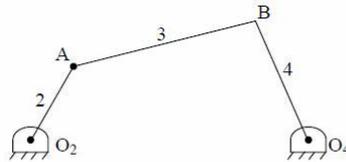
Absolute velocity: Velocity of a point with respect to a fixed point (zero velocity point).



$$V_a = \omega_2 \times r$$

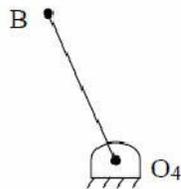
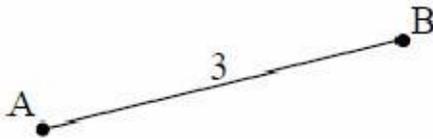
$$V_a = \omega_2 \times O_2 A$$

Relative velocity: Velocity of a point with respect to another point 'x'



Ex: $V_{ba} \rightarrow$ Velocity of point B with respect to A

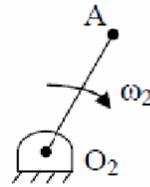
$V_{ba} = V_{ab}$ Equal in magnitude but opposite in direction



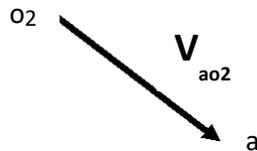
V_b Absolute velocity is velocity of B with respect to O_4 (fixed point, zero velocity point)

Note: Capital letters are used for configuration diagram. Small letters are used for velocity vector diagram.

Velocity Vector :



Velocity	Line of action	Direction	magnitude
V_{ao2} (ω_{2a})	Perpendicular to OA through 'o'		$(\omega_{O2A} \times O2A)$



Velocity analysis of any mechanism can be carried by various methods.

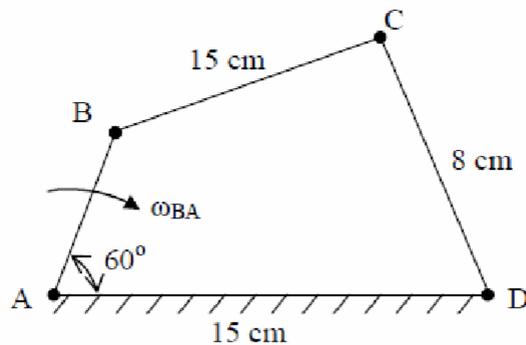
1. By graphical method
2. By relative velocity method
3. By instantaneous method

□ **Graphical Method**

The following points are to be considered while solving problems by this method.

1. Draw the configuration design to a suitable scale.
2. Locate all fixed point in a mechanism as a common point in velocity diagram.
3. Choose a suitable scale for the vector diagram velocity.
4. The velocity vector of each rotating link is \perp to the link.
5. Velocity of each link in mechanism has both magnitude and direction. Start from a point whose magnitude and direction is known.
6. The points of the velocity diagram are indicated by small letters.

1. Four – Bar Mechanism: In a four bar chain ABCD link AD is fixed and is 15 cm long. The crank AB is 4 cm long rotates at 180 rpm (cw) while link CD rotates about D is 8 cm long BC = AD and $\angle BAD = 60^\circ$. Find angular velocity of link CD.



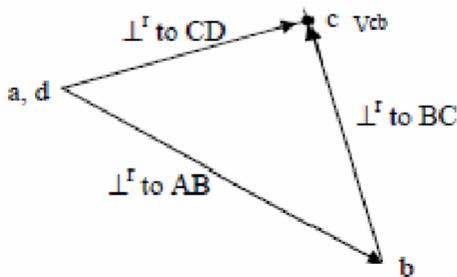
Configuration Diagram

Velocity vector diagram

$$V_b = \omega r = \omega_{ba} \times AB = \frac{2\pi \times 120}{60} \times 4 = 50.24 \text{ cm/sec}$$

Choose a suitable scale

$$1 \text{ cm} = 20 \text{ m/s} = \overline{ab}$$



$$V_{cb} = \overline{bc}$$

$$V_c = \overline{dc} = 38 \text{ cm/sec} = V_{cd}$$

We know that $V = \omega R$

$$V_{cd} = \omega_{CD} \times CD$$

$$\omega_{CD} = \frac{V_{cd}}{CD} = \frac{38}{8} = 4.75 \text{ rad/sec (cw)}$$

Velocity of Rubbing:

Two links of a mechanism having turning point will be connected by pins. When the links are motion they rub against pin surface. The velocity of rubbing of pins depends on the angular velocity of links relative to each other as well as direction.

For example: In a four bar mechanism we have pins at points A, B, C and D.

$$V_{ra} = \omega_{ab} \times \text{radius of pin A (rpa)}$$

+ sign is used ω_{ab} is CW and ω_{bc} is CCW i.e. when angular velocities are in opposite directions use + sign when angular velocities are in same directions use -ve sign.

$$V_{rb} = (\omega_{ab} + \omega_{bc}) \text{ radius rpb}$$

$$V_{rC} = (\omega_{bc} + \omega_{cd}) \text{ radius rpc}$$

$$V_{rD} = \omega_{cd} \text{ rpd}$$

Instantaneous Method

To explain instantaneous centre let us consider a plane body P having a non-linear motion relative to another body q consider two points A and B on body P having velocities as V_a and V_b respectively in the direction shown.

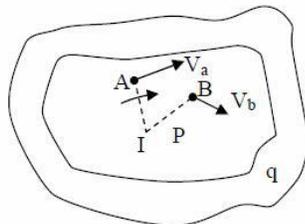


Fig. 1

If a line is drawn \perp to V_a , at A the body can be imagined to rotate about some point on the line. Thirdly, centre of rotation of the body also lies on a line \perp to the direction of V_b at B. If the intersection of the two lines is at I, the body P will be rotating about I at that instant. The point I is known as the instantaneous centre of rotation for the body P. The position of instantaneous centre changes with the motion of the body.

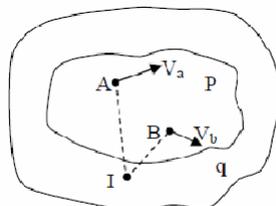


Fig. 2

In case of the \perp lines drawn from A and B meet outside the body P as shown in Fig 2.

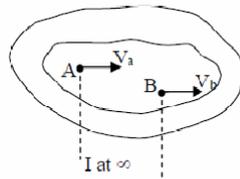


Fig. 3

If the direction of V_a and V_b are parallel to the r at A and B met at ∞ . This is the case when the body has linear motion

□ **Number of Instantaneous Centers**

The number of instantaneous centers in a mechanism depends upon number of links. If N is the number of instantaneous centers and n is the number of links.

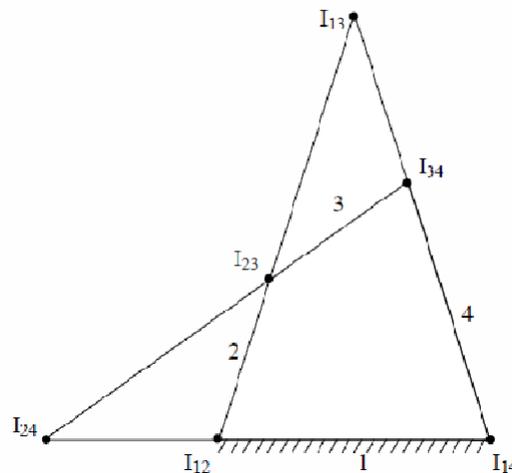
$$N = \frac{n(n-1)}{2}$$

□ **Types of Instantaneous Centers**

There are three types of instantaneous centers namely fixed, permanent and neither fixed nor permanent.

Example: Four bar mechanism. $n = 4$.

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



Fixed instantaneous center I_{12}, I_{14}

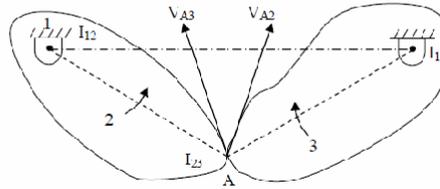
Permanent instantaneous center I_{23}, I_{34}

Neither fixed nor permanent instantaneous center I_{13}, I_{24}

□ **Arnold Kennedy theorem of three centers:**

STATEMENT: If three bodies have motion relative to each other, their instantaneous centers should lie in a straight line.

Proof:



Consider a three link mechanism with link 1 being fixed link 2 rotating about I_{12} and link 3 rotating about I_{13} . Hence, I_{12} and I_{13} are the instantaneous centers for link 2 and link 3. Let us assume that instantaneous center of link 2 and 3 be at point A i.e. I_{23} . Point A is a coincident point on link 2 and link 3.

Considering A on link 2, velocity of A with respect to I_{12} will be a vector V_{A2} \perp to link A I_{12} . Similarly for point A on link 3, velocity of A with respect to I_{13} will be \perp to A I_{13} . It is seen that velocity vector of V_{A2} and V_{A3} are in different directions which is impossible. Hence, the instantaneous center of the two links cannot be at the assumed position.

It can be seen that when I_{23} lies on the line joining I_{12} and I_{13} the V_{A2} and V_{A3} will be same in magnitude and direction. Hence, for the three links to be in relative motion all the three centers should lie in a same straight line.

Steps to locate instantaneous centers:

Step 1: Draw the configuration diagram.

Step 2: Identify the number of instantaneous centers

Step 3: Identify the instantaneous centers by circle diagram.

Step 4: Locate all the instantaneous centers by making use of Kennedy's theorem.

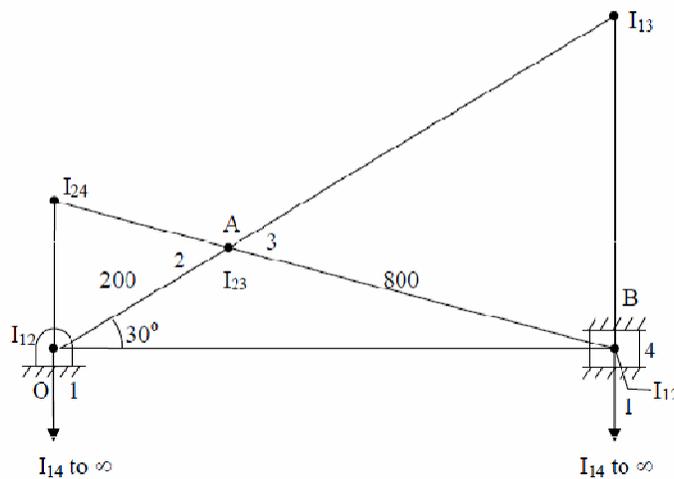
Eg :A slider crank mechanism has lengths of crank and connecting rod equal to 200 mm and 200 mm respectively locate all the instantaneous centers of the mechanism for the position of the crank when it has turned through 30° from IOC. Also find velocity of slider and angular velocity of connecting rod if crank rotates at 40 rad/sec.

Step 1: Draw configuration diagram to a suitable scale.

Step 2: Determine the number of links in the mechanism and find number of instantaneous centers.

$$N = \frac{(n-1)n}{2}$$

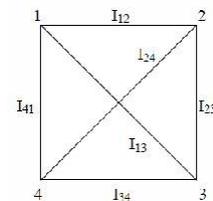
$$n = 4 \text{ links} \quad N = \frac{4(4-1)}{2} = 6$$



Step 3: Identify instantaneous centers.

- Suit it is a 4-bar link the resulting figure will be a square.

Locate fixed and permanent instantaneous centers. To locate neither fixed nor permanent instantaneous centers use Kennedy's three centers theorem



Step 4: Velocity of different points.

$$V_a = \omega_2 AI_{12} = 40 \times 0.2 = 8 \text{ m/s}$$

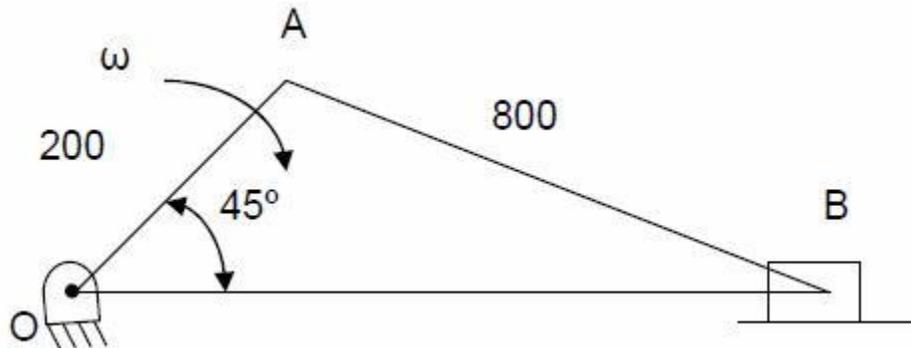
$$\text{also } V_a = \omega_3 \times AI_{13}$$

$$\therefore \omega_3 = \frac{V_a}{AI_{13}}$$

$$V_b = \omega_3 \times BI_{13} = \text{Velocity of slider.}$$

KLEIN'S Construction:

This method helps us to draw the velocity and acceleration diagrams on the construction diagram itself. The crank of the configuration diagram represents the velocity and acceleration line of the moving end (crank).

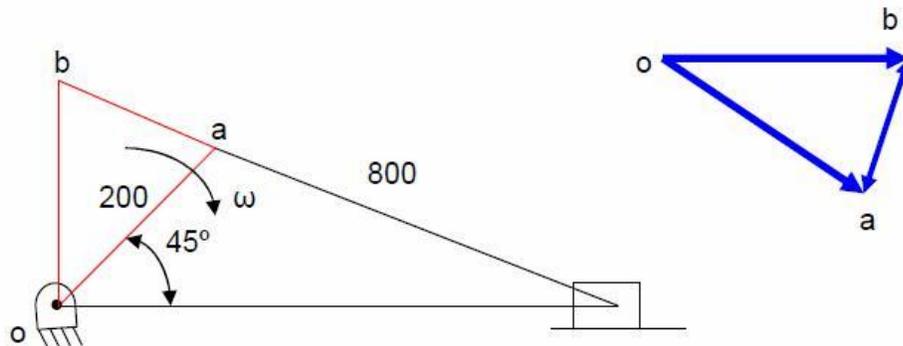


To draw the velocity vector diagram:

Link OA represents the velocity vector of A with respect to O.

$$V_{Oa} = oa = \omega r = \omega OA.$$

Draw a line perpendicular at O, extend the line BA to meet this perpendicular line at b. oab is the velocity vector diagram rotated through 90° opposite to the rotation of the crank.



Acceleration diagram:

The line representing Crank OA represents the acceleration of A with respect to O. To draw the acceleration diagram follow the steps given below.

KINEMATICS OF MACHINES

UNIT - III

ACCELERATION ANALYSIS

Acceleration Analysis :

Rate of change of velocity is acceleration. A change in velocity requires any one of the following conditions to be fulfilled:

- Change in magnitude only
- Change in direction only
- Change in both magnitude and direction

When the velocity of a particle changes in magnitude and direction it has two component of acceleration.

1. Radial or centripetal acceleration

$$f^c = \omega^2 r$$

Acceleration is parallel to the link and acting towards centre.

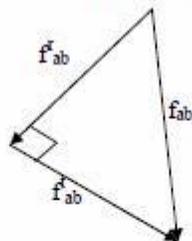
2. Tangential Acceleration:

$$F^T = \alpha R$$



$$f_{ab}^c = \omega^2 AB$$

Acts parallel to BA and acts from B to A.



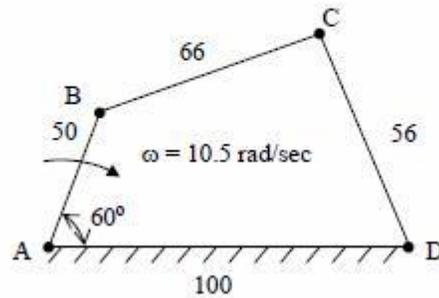
$f^t = \alpha BA$ acts \perp^T to link.

$$f_{BA} = f_{BA}^c + f_{BA}^t$$

Acceleration	Component	Line of action	Direction	Magnitude
f_{ba}	Centripetal $f_{ba}^c (b_1^1 a_1)$	parallel to BA through a_1	From B toward A in link	V_{BA}^2 / BA
	Tangential $f_{ba}^t (b_1^1 b_1)$	perpendicular to BA through end point of f_{ba}^c (ie b_1^1)	Depends on the sense of α_{BA}	$\alpha_{BA} \times BA$

- **Problem 1:** Four bar mechanism. For a 4-bar mechanism shown in figure draw velocity and acceleration diagram.

All dimensions
are in mm



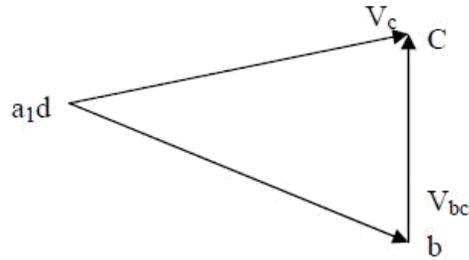
Step 1: Draw configuration diagram to a scale.

Step 2: Draw velocity vector diagram to a scale.

$$V_b = \omega_2 \times AB$$

$$V_b = 10.5 \times 0.05$$

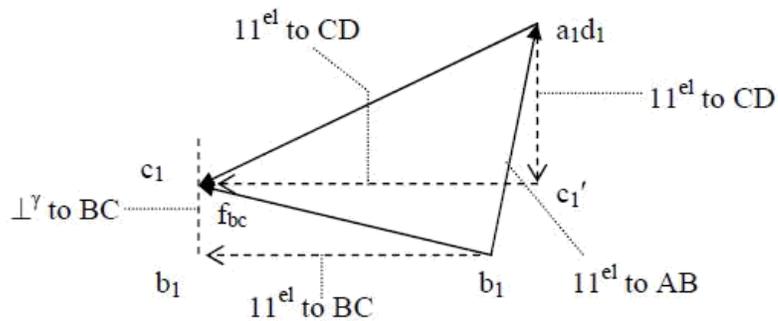
$$V_b = 0.525 \text{ m/s}$$



Step 3: Prepare a table as shown below:

Sl. No.	Link	Magnitude	Direction	Sense
1.	AB	$f^c = \omega_{AB}^2 r$ $f^c = (10.5)^2 / 0.525$ $f^c = 5.51 \text{ m/s}^2$	Parallel to AB	\rightarrow A
2.	BC	$f^c = \omega_{BC}^2 r$ $f^c = 1.75$ $f^t = \alpha r$	Parallel to BC \perp^r to BC	\rightarrow B -
3.	CD	$f^c = \omega_{CD}^2 r$ $f^c = 2.75$ $f^t = ?$	Parallel to DC \perp^r to DC	\rightarrow D -

Step 4: Draw the acceleration diagram.



- Choose a suitable scale to draw acceleration diagram.
- Mark the zero acceleration point a_1d_1 .
- Link AB has only centripetal acceleration. Therefore, draw a line parallel to AB and toward A from a_1d_1 equal to 5.51 m/s^2 i.e. point b_1 .
- From b_1 draw a vector parallel to BC points towards B equal to 1.75 m/s^2 (b_1^1).
- From b_1^1 draw a line \perp^f to BC. The magnitude is not known.
- From a_1d_1 draw a vector parallel to AD and pointing towards D equal to 2.72 m/s^2 i.e. point c_1 .
- From c_1^1 draw a line \perp^f to CD to intersect the line drawn \perp^f to BC at c_1 ,
 $\overline{d_1c_1} = f_{CD}$ and $\overline{b_1c_1} = f_{bc}$.

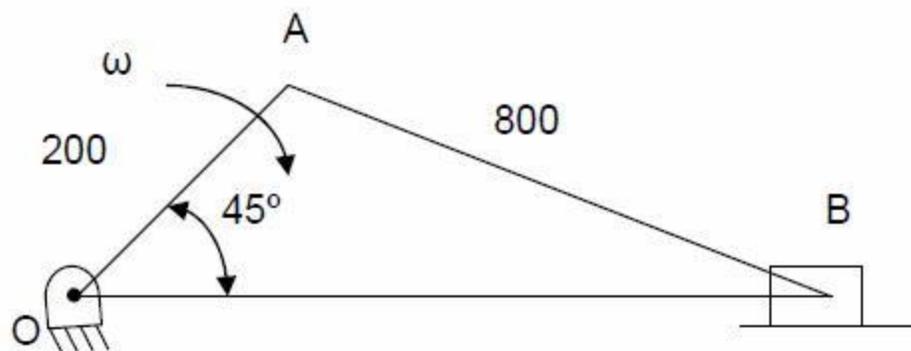
To determine angular acceleration.

$$\alpha_{BC} = \frac{f_{bc}^t}{BC} = \frac{\overline{c_1b_1^1}}{BC} = 34.09 \text{ rad/sec (CCW)}$$

$$\alpha_{CD} = \frac{f_{cd}^t}{CD} = \frac{\overline{c_1c_1^1}}{CD} = 79.11 \text{ rad/sec (CCW)}$$

KLEIN'S Construction:

This method helps us to draw the velocity and acceleration diagrams on the construction diagram itself. The crank of the configuration diagram represents the velocity and acceleration line of the moving end (crank).

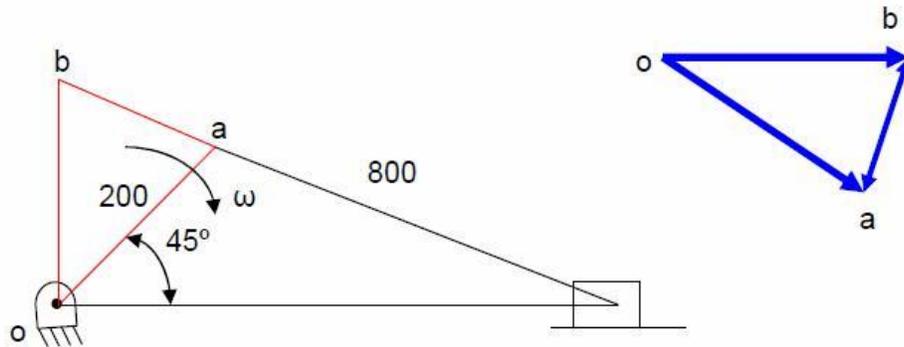


To draw the velocity vector diagram:

Link OA represents the velocity vector of A with respect to O.

$$V_{oa} = oa = \omega r = \omega OA.$$

Draw a line perpendicular at O, extend the line BA to meet this perpendicular line at b. oab is the velocity vector diagram rotated through 90° opposite to the rotation of the crank.

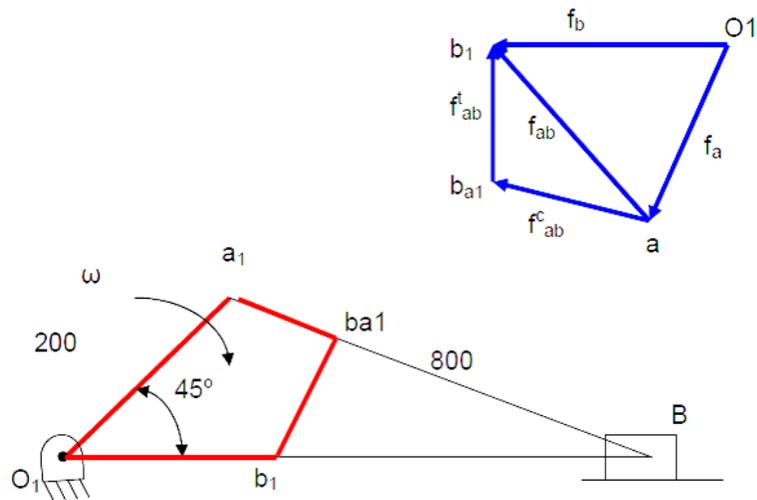


Acceleration diagram:

The line representing Crank OA represents the acceleration of A with respect to O. To draw the acceleration diagram follow the steps given below.

- Draw a circle with OA as radius and A as centre.
- Draw another circle with AB as diameter.
- The two circles intersect each other at two points C and D.
- Join C and D to meet OB at b_1 and AB at E.

O_1, a_1, b_1 and b_1 is the required acceleration diagram rotated through 180° .



Synthesis of Mechanisms :

kinematic synthesis means the design or creation of a mechanism to attain specific motion characteristics. In this sense synthesis is the inverse problem of analysis. Synthesis is the very essence of design because it represents the creation of new hardware to meet particular requirements of motion: displacement; velocity; acceleration; individually or in combination.

1. Type Synthesis

It deals with the selection of type of mechanism to attain the required motion characteristics.

2. Number Synthesis:

Number synthesis is the second step in the process of mechanism design. It deals with determining the number of DOF and the number of links and joints required.

3. Dimensional Synthesis:

The third step in mechanism design is dimensional synthesis. This involves determination of link lengths

Unit IV CAMS

Introduction : A *cam* is a rotating machine element which gives reciprocating or oscillating motion to another element known as *follower*. The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam.

The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc.

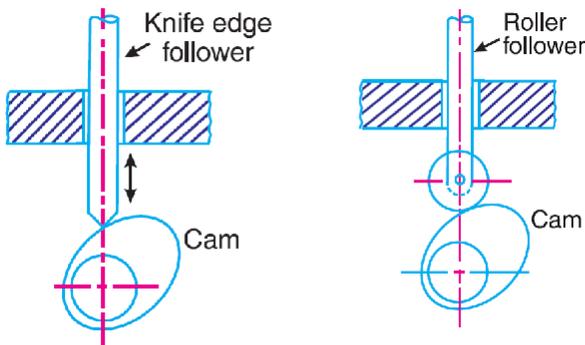
Classification of Followers :

The followers may be classified as discussed below :

1. According to the surface in contact.

The followers, according to the surface in contact, are as follows :

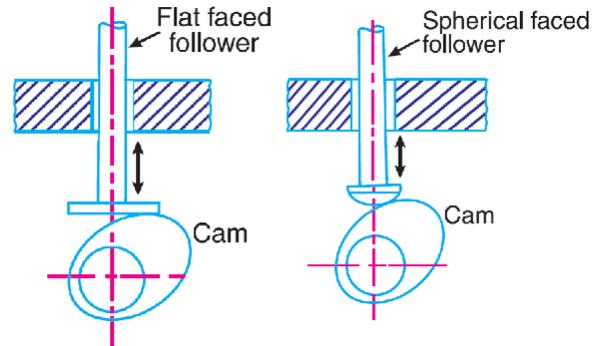
(a) **Knife edge follower.** When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower. In knife edge followers, a considerable side thrust exists between the follower and the guide.



(b) **Roller follower.** When the contacting end of the follower is a roller, it is called a roller Follower . Since the rolling motion takes place between the contacting surfaces (*i.e.* the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide.

(c) **Flat faced or mushroom follower.** When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower. The side thrust between the follower and the guide is much reduced in case of flat faced followers. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axis of the follower. The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

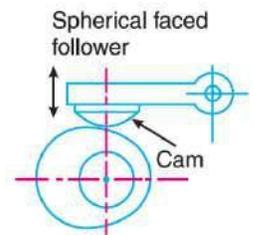
(d) **Spherical faced follower.** When the contacting end of the follower is of spherical shape, it is called a spherical faced follower



2. According to the motion of the follower. The followers, according to its motion, are of the following two types:

(a) **Reciprocating or translating follower.** When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown above are all reciprocating or translating followers.

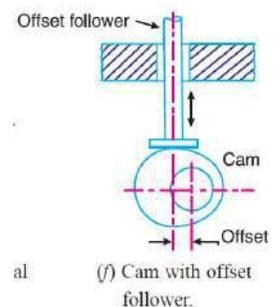
(b) **Oscillating or rotating follower.** When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower.



3. According to the path of motion of the follower. The followers, according to its path of motion, are of the following two types:

(a) **Radial follower.** When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower. The followers, as shown above are all radial followers.

(b) **Off-set follower.** When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower.



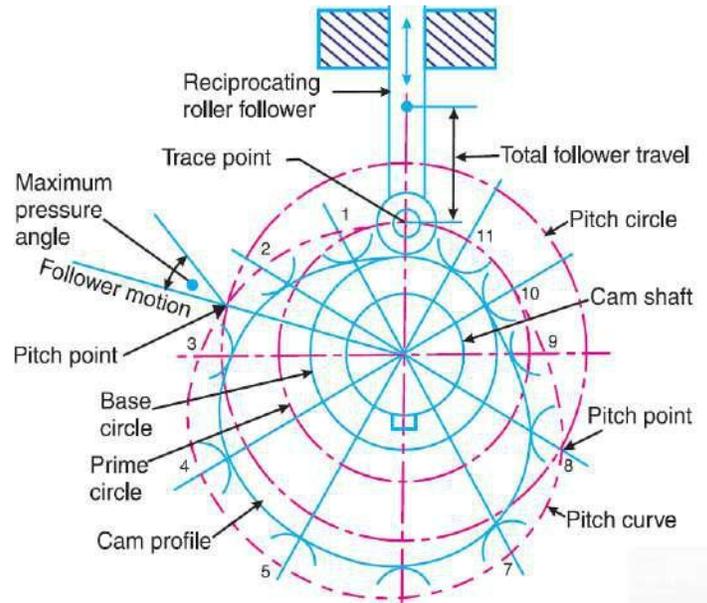
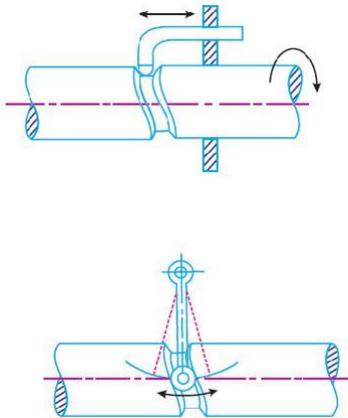
Classification of Cams

Though the cams may be classified in many ways, yet the following two types are important from the subject point of view

1. Radial or disc cam. In radial cams, the follower reciprocates or oscillates in a direction perpendicular to

the cam axis. The cams as shown above are all radial cams.

2. Cylindrical cam. In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface.



Terms Used in Radial Cams

Fig. shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. Base circle. It is the smallest circle that can be drawn to the cam profile.

2. Trace point. It is a reference point on the follower and is used to generate the *pitch curve*.

In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.

3. Pressure angle. It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. Pitch point. It is a point on the pitch curve having the maximum pressure angle.

5. Pitch circle. It is a circle drawn from the centre of the cam through the pitch points.

6. Pitch curve. It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

7. Prime circle. It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

8. Lift or stroke. It is the maximum travel of the follower from its lowest position to the topmost position.

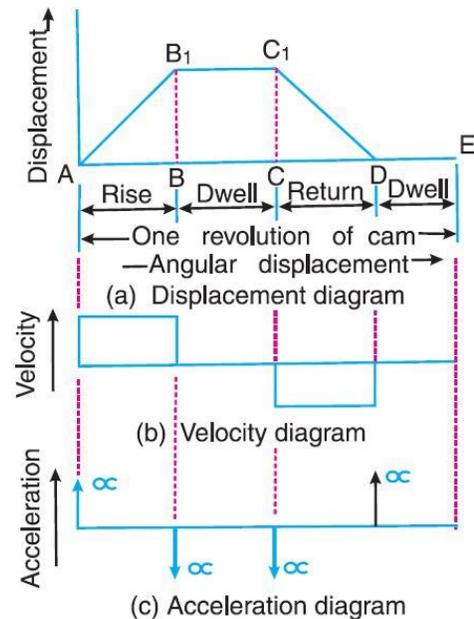
Motion of the Follower

The follower, during its travel, may have one of the following motions.

1. Uniform velocity
2. Simple harmonic motion
3. Uniform acceleration and retardation and
4. Cycloidal motion

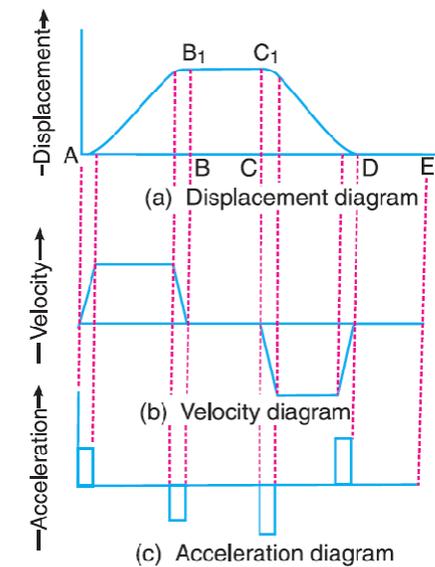
1. Follower Moves with Uniform Velocity

The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown. The abscissa (base) represents the time (*i.e.* the number of seconds required for the cam to complete one revolution) or it may represent the angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of the follower.



Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB and CD must be straight lines. A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known as **dwelling periods**, as shown by lines B_1C_1 and DE . The acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however, impracticable.

In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower. This may be done by rounding off the sharp corners of the displacement diagram at the beginning and at the end of each stroke. By doing so, the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke.



2. Follower Moves with Simple Harmonic Motion

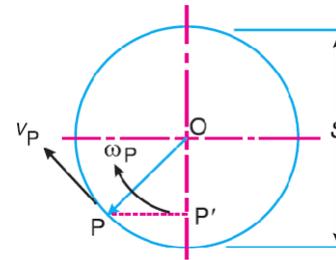
The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve. The velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.

S = Stroke of the follower

θ_O θ_R = Angular displacement of the cam during outstroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

Consider a point P moving at a uniform speed ω radians per sec round the circumference of a circle with the stroke S as diameter. The point P' (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P' .



Peripheral speed of the point P

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$

and maximum velocity of the follower on the outstroke,

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega S}{2 \theta_O}$$

that the centripetal acceleration of the point P , Maximum acceleration of the follower on the outstroke,

$$a_O = a_P = \frac{\pi^2 \omega^2 S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \omega S}{2 \theta_R}$$

and maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 S}{2(\theta_R)^2}$$

3. Follower Moves with Uniform Acceleration and Retardation

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown.

We know that time required for the follower during outstroke, $t_O = \theta_O / \omega$

and time required for the follower during return stroke, $t_R = \theta_R / \omega$

Mean velocity of the follower during outstroke = S/t_O

mean velocity of follower during return stroke = S/t_R

Since the maximum velocity of follower is equal to twice the mean velocity, therefore maximum velocity of the follower during outstroke,

$$v_O = \frac{2S}{t_O} = \frac{2\omega \cdot S}{\theta_O}$$

Similarly, maximum velocity of the follower during return stroke,

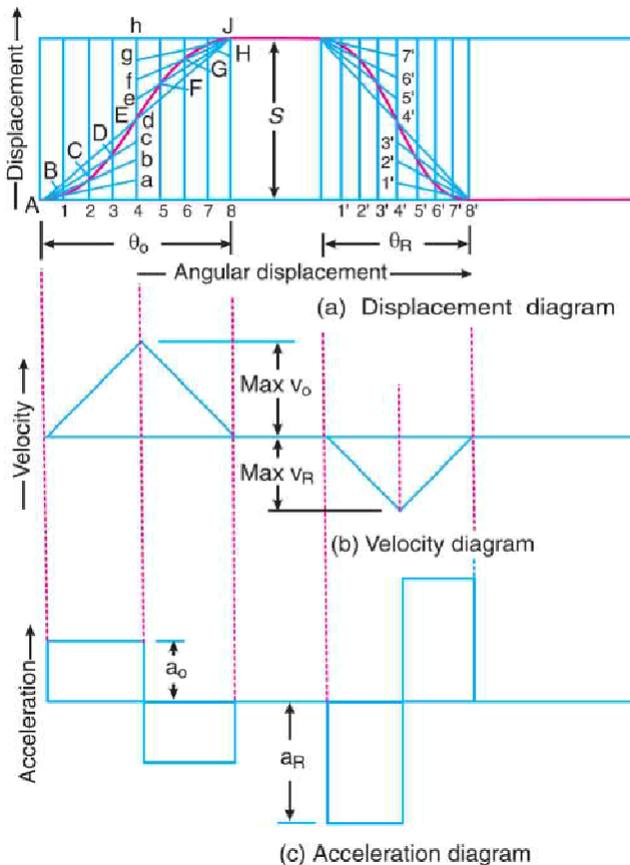
$$v_R = \frac{2\omega \cdot S}{\theta_R}$$

During first half of the outstroke there is uniform acceleration and during the second half of the out stroke there is uniform retardation. Thus, the maximum velocity of the follower is reached after the time $t_O / 2$ (during out stroke) and $t_R / 2$ (during return stroke). Maximum acceleration of the follower during outstroke,

$$a_O = \frac{v_O}{t_O/2} = \frac{2 \times 2\omega \cdot S}{t_O \cdot \theta_O} = \frac{4\omega^2 \cdot S}{(\theta_O)^2}$$

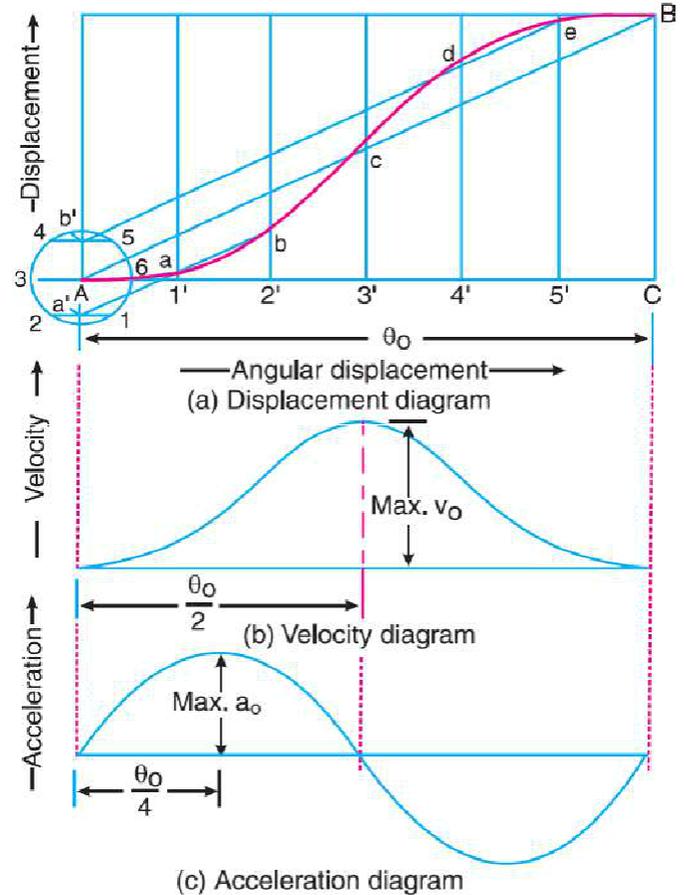
Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\omega^2 \cdot S}{(\theta_R)^2}$$



4. Follower Moves with Cycloidal Motion

cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line. In case of cams, this straight line is a stroke of the follower which is translating and the circumference of the rolling circle is equal to the stroke (S) of the follower. Therefore the radius of the rolling circle is $S / 2\pi$



Let θ = Angle through which the cam rotates in time t seconds, and ω = Angular velocity of the cam. We know that displacement of the follower after time t seconds,

$$x = S \left[\frac{\theta}{\theta_O} - \frac{1}{2\pi} \sin \left(\frac{2\pi\theta}{\theta_O} \right) \right]$$

Velocity of the follower after time t seconds,

$$\begin{aligned} \frac{dx}{dt} &= S \left[\frac{1}{\theta_O} \times \frac{d\theta}{dt} - \frac{2\pi}{2\pi\theta_O} \cos \left(\frac{2\pi\theta}{\theta_O} \right) \frac{d\theta}{dt} \right] \\ &= \frac{S}{\theta_O} \times \frac{d\theta}{dt} \left[1 - \cos \left(\frac{2\pi\theta}{\theta_O} \right) \right] = \frac{\omega S}{\theta_O} \left[1 - \cos \left(\frac{2\pi\theta}{\theta_O} \right) \right] \end{aligned}$$

The velocity is maximum, when

$$\cos\left(\frac{2\pi\theta}{\theta_0}\right) = -1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_0} = \pi \quad \text{or} \quad \theta = \theta_0 / 2$$

Substituting $\theta = \theta_0 / 2$ we have maximum velocity of the follower during outstroke,

$$v_O = \frac{\omega.S}{\theta_0}(1+1) = \frac{2\omega.S}{\theta_0}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega.S}{\theta_R}$$

Now, acceleration of the follower after time t sec,

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{\omega.S}{\theta_0} \left[\frac{2\pi}{\theta_0} \sin\left(\frac{2\pi\theta}{\theta_0}\right) \frac{d\theta}{dt} \right] \\ &= \frac{2\pi\omega^2.S}{(\theta_0)^2} \sin\left(\frac{2\pi\theta}{\theta_0}\right) \end{aligned}$$

The acceleration is maximum, when

$$\sin\left(\frac{2\pi\theta}{\theta_0}\right) = 1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_0} = \frac{\pi}{2} \quad \text{or} \quad \theta = \theta_0 / 4$$

Substituting $\theta = \theta_0 / 4$ we have maximum acceleration of the follower during outstroke,

$$a_O = \frac{2\pi\omega^2.S}{(\theta_0)^2}$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{2\pi\omega^2.S}{(\theta_R)^2}$$

In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

In constructing the cam profile, the principle of kinematic inversion is used, i.e. the cam is imagined to be stationary and the follower is allowed to rotate in the **opposite direction** to the **cam rotation**.

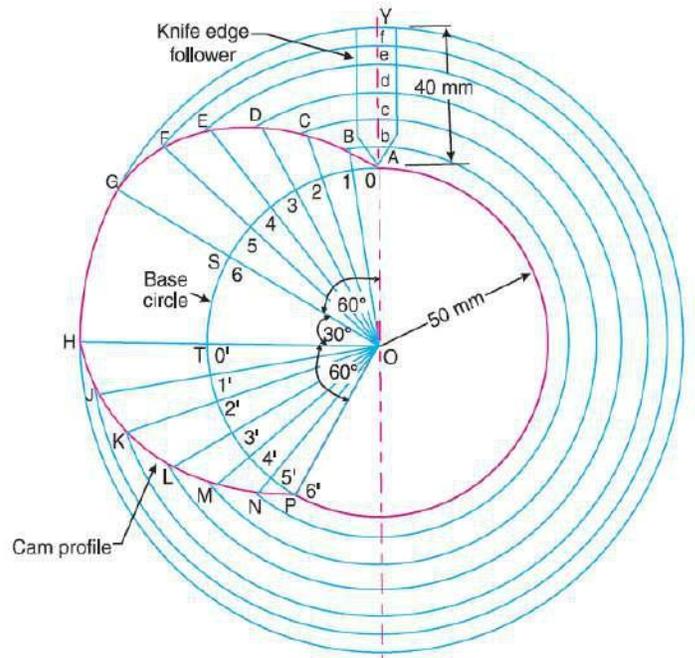
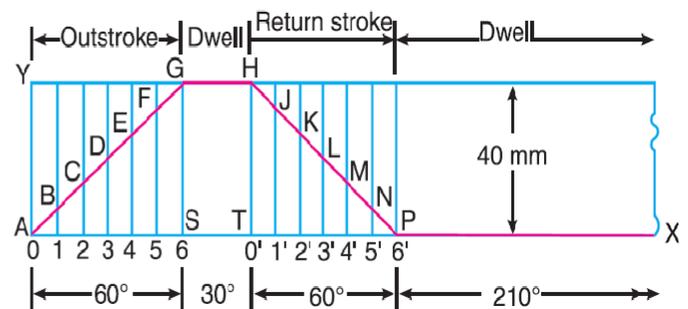
Case (i) Knife edge follower without offset :

A cam is to give the following motion to a knife-edged follower :

1. Outstroke during 60° of cam rotation ;
2. Dwell for the next 30° of cam rotation ;
3. Return stroke during next 60° of cam rotation, and
4. Dwell for the remaining 210° of cam rotation.

The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam **when the axis of the follower passes through the axis of the cam shaft**,

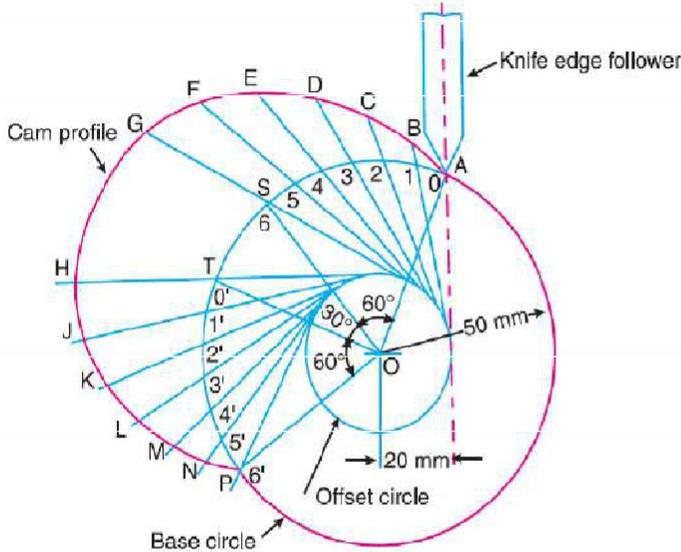
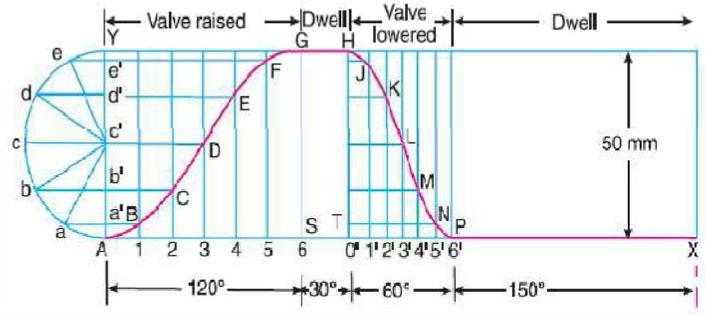
Sol :



Construction of Cam Profile for a Radial Cam

Case (ii) Knife edge follower with offset :

For the above problem in case (i) , if the follower is **offset by 20 mm** from the cam axis , then there is no change in the displacement diagram . But the construction of cam profile changes.



Case (iii) Roller follower with and without offset :

An example problem is solved considering the roller follower with and without offset as two cases. Also it is asked to find the maximum acceleration and to draw the variation of displacement , velocity and acceleration

A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a **roller follower**, at the end of a valve rod, motion described below :

1. To raise the valve through 50 mm during 120° rotation of the cam ;
2. To keep the valve fully raised through next 30°;
3. To lower the valve during next 60°; and
4. To keep the valve closed during rest of the revolution i.e. 150° ;

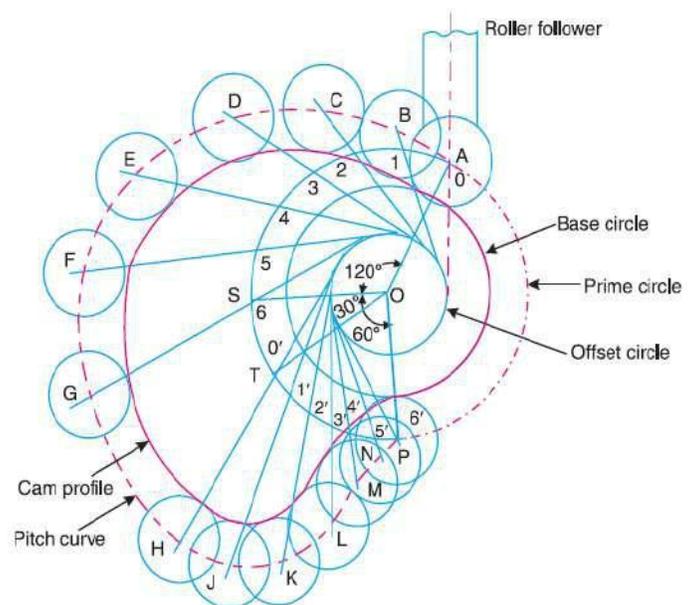
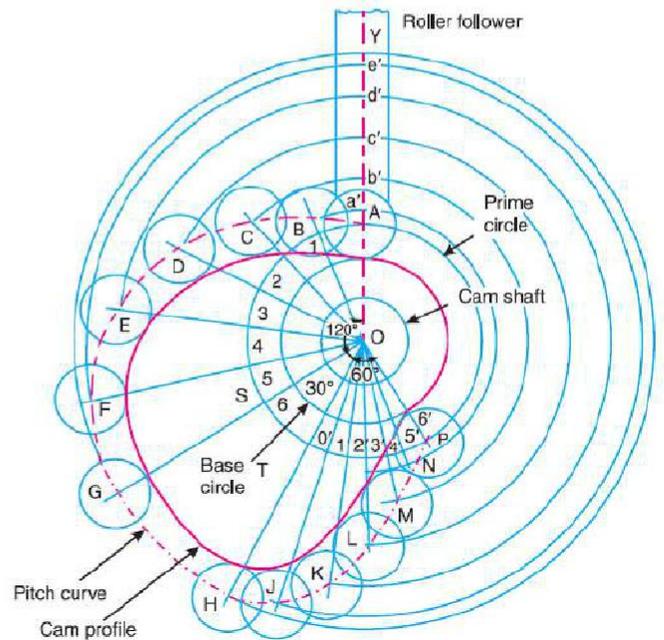
The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm.

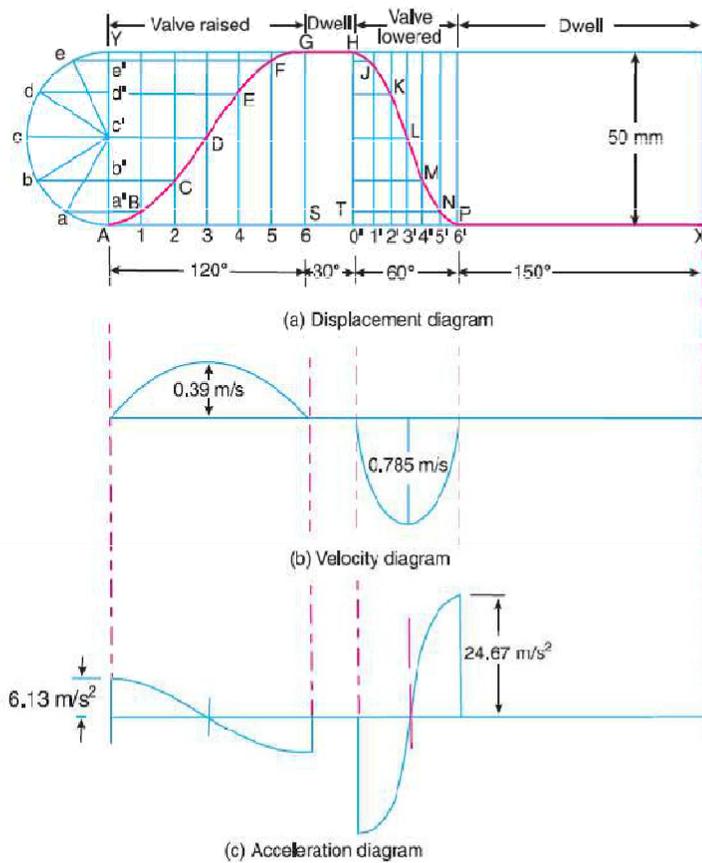
Draw the profile of the cam when

- (a) the line of stroke of the valve rod passes through the axis of the cam shaft, and
- (b) the line of the stroke is offset 15 mm from the axis of the cam shaft.

The displacement of the valve, while being raised and lowered, is to take place **with simple harmonic motion**. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m.

Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam.

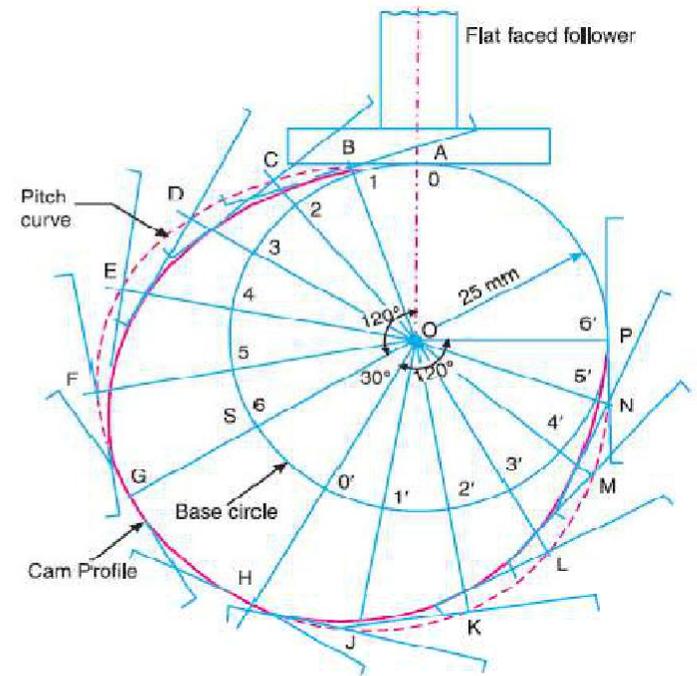




and maximum acceleration of the valve rod to lower the valve

$$a_R = \frac{\pi^2 \omega^2 \cdot S}{2(\theta_R)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(1.047)^2} = 24.67 \text{ m/s}^2$$

Case (iv) Flat faced follower without offset



We know that angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

We also know that maximum velocity of the valve rod to raise valve,

$$v_O = \frac{\pi \omega \cdot S}{2\theta_O} = \frac{\pi \times 10.47 \times 0.05}{2 \times 2.1} = 0.39 \text{ m/s}$$

and maximum velocity of the valve rod to lower the valve,

$$v_R = \frac{\pi \omega \cdot S}{2\theta_R} = \frac{\pi \times 10.47 \times 0.05}{2 \times 1.047} = 0.785 \text{ m/s}$$

The velocity diagram for one complete revolution of the cam is shown

We know that the maximum acceleration of the valve rod to raise the valve,

$$a_O = \frac{\pi^2 \omega^2 \cdot S}{2(\theta_O)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(2.1)^2} = 6.13 \text{ m/s}^2$$

Case (v) Oscillating follower

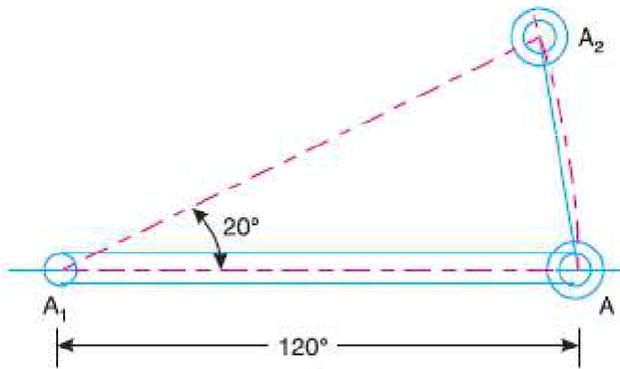
Draw a cam profile to drive an oscillating roller follower to the specifications given below :

(a) Follower to move outwards through an angular displacement of 20° during the first 120° rotation of the cam ;

(b) Follower to return to its initial position during next 120° rotation of the cam ;

(c) Follower to dwell during the next 120° of cam rotation.

The distance between pivot centre and roller centre = 120 mm ; distance between pivot centre and cam axis = 130 mm ; minimum radius of cam = 40 mm ; radius of roller = 10 mm ; inward and outward strokes take place with simple harmonic motion.

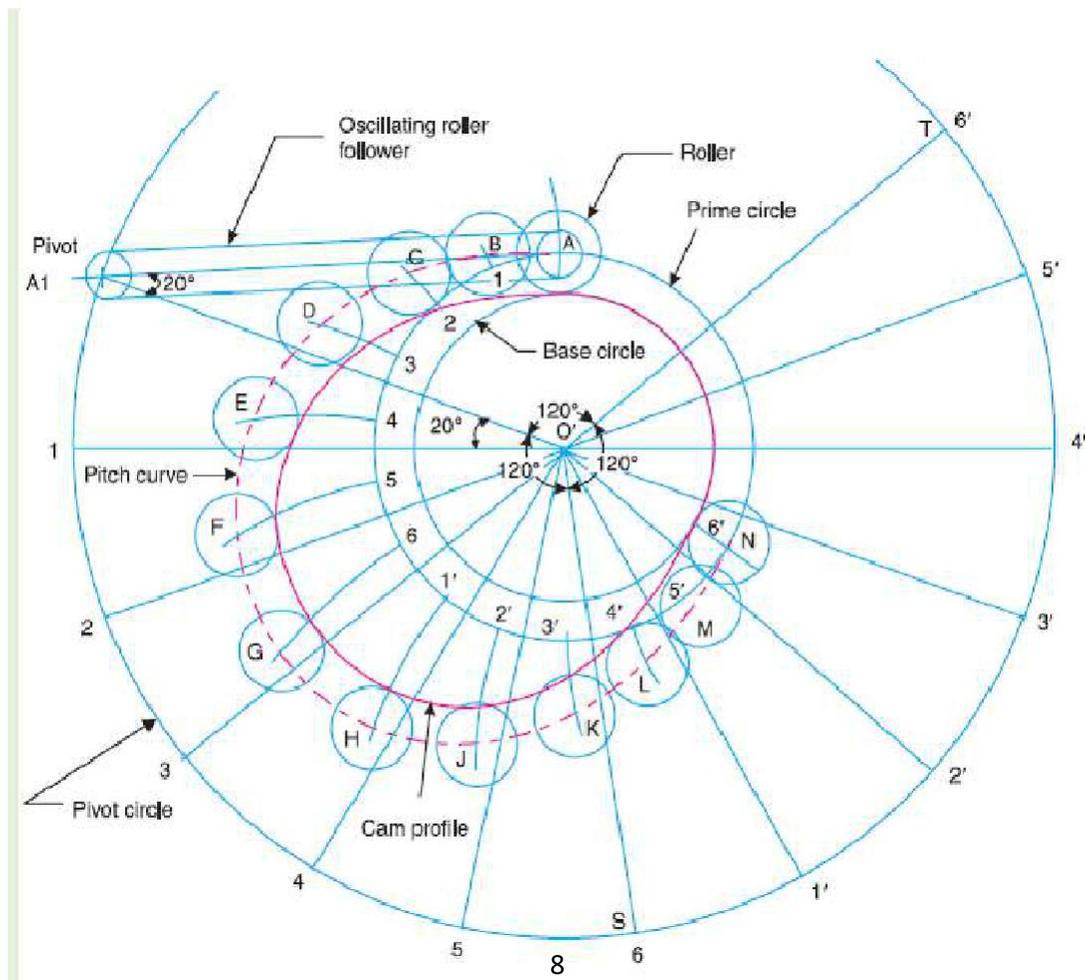


We know that the angular displacement of the roller follower = $20^\circ = 20 \times \frac{\pi}{180} = \frac{\pi}{9}$ rad

Since the distance between the pivot centre and the roller centre (i.e. the radius $A_1 A$) is 120 mm, therefore length of the arc AA_2 , along which the displacement of the roller actually takes place = $120 \times \frac{\pi}{9} = 41.88$ mm

(∵ Length of arc = Radius of arc \times Angle subtended by the arc at the centre in radians)

Since the angle is very small, therefore length of chord AA_2 is taken equal to the length of arc AA_2 . Thus in order to draw the displacement diagram, we shall take lift of the follower equal to length of chord AA_2 i.e. 41.88 mm.



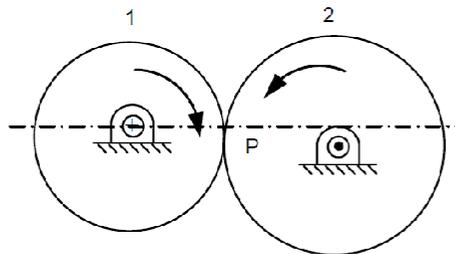
UNIT V
Part-B
GEAR TRAINS

A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. They often consist of multiple gears in the train.

Types of Gear trains :

1. Simple Gear Train

In this gear train, there are series of gears which are capable of receiving and transmitting motion from one gear to another. They may mesh externally or internally. Each gear rotates about separate axis fixed to the frame. Figure shows two gears in external meshing. Let t_1, t_2 be number of teeth on gears 1 and 2.

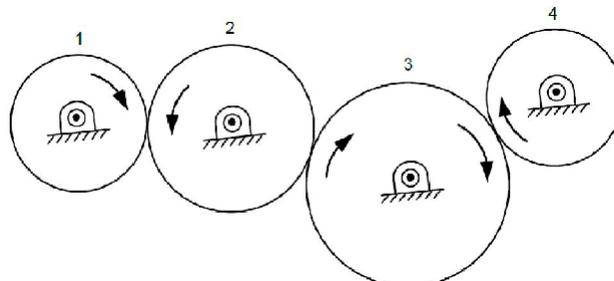


Let N_1, N_2 be speed in rpm for gears 1 and 2. The velocity of P ,

$$V_P = \frac{2\pi N_1 d_1}{60} = \frac{2\pi N_2 d_2}{60}$$

$$\therefore \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{t_2}{t_1}$$

The two meshing gears in external meshing rotate in opposite sense whereas in internal meshing they rotate in same sense. In simple gear train, there can be more than two gears also as shown in Figure



Let N_1, N_2, N_3, \dots be speed in rpm of gears 1, 2, 3, \dots etc., and t_1, t_2, t_3, \dots be number of teeth of respective gears 1, 2, 3, \dots , etc.

In this gear train, gear 1 is input gear, gear 4 is output gear and gears 2, 3 are intermediate gears.

The gear ratio of the gear train is give by

$$\text{Gear Ratio} = \frac{N_1}{N_4} = \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4}$$

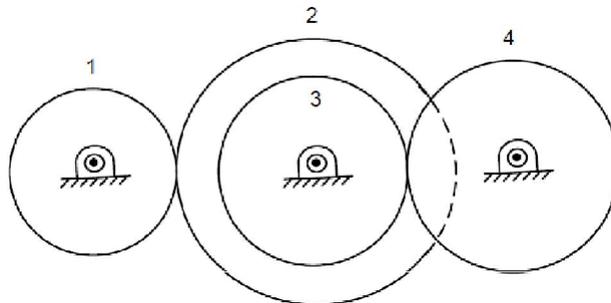
$$\therefore \frac{N_1}{N_2} = \frac{t_2}{t_1}; \quad \frac{N_2}{N_3} = \frac{t_3}{t_2} \quad \text{and} \quad \frac{N_3}{N_4} = \frac{t_4}{t_3}$$

$$\text{Therefore,} \quad \frac{N_1}{N_4} = \frac{t_2}{t_1} \times \frac{t_3}{t_2} \times \frac{t_4}{t_3} = \frac{t_4}{t_1}$$

This expression indicates that the intermediate gears have no effect on gear ratio. These intermediate gears fill the space between input and output gears and have effect on the sense of rotation of output gear

2. Compound Gear Train

In this type of gear train, at least two gears are mounted on the same shaft and they rotate at the same speed. This gear train is shown in Figure 3.30 where gears 2 and 3 are mounted on same shaft and they rotate at the same speed, i.e. N_2, N_3



Let N_1, N_2, N_3, \dots be speed in rpm of gears 1, 2, 3, \dots , etc. and t_1, t_2, t_3, \dots , etc. be number of teeth of respective gears 1, 2, 3, \dots , etc.

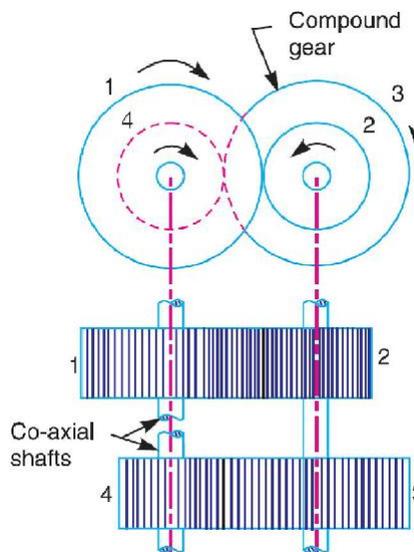
$$\begin{aligned} \text{Gear Ratio} &= \frac{N_1}{N_4} = \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4} \\ &= \frac{t_2}{t_1} \times \frac{t_4}{t_3} \end{aligned}$$

Therefore, unlike simple gear train the gear ratio is contributed by all the gears. This gear train is used in conventional automobile gear box.

3. Reverted Gear Train:

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train as shown in Fig. We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is like

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$



Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same

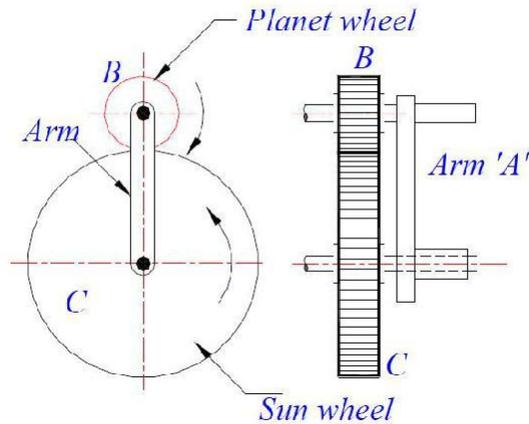
$$r_1 + r_2 = r_3 + r_4$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius

$$T_1 + T_2 = T_3 + T_4$$

4. Epicyclic gear train:

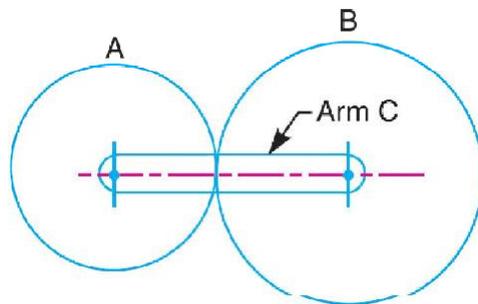
Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun. Here is a picture of a typical gear box. This design can produce large gear ratios in a small space and are used on a wide range of applications from marine gearboxes to electric screwdrivers.



The diagram shows a gear B on the end of an arm. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.

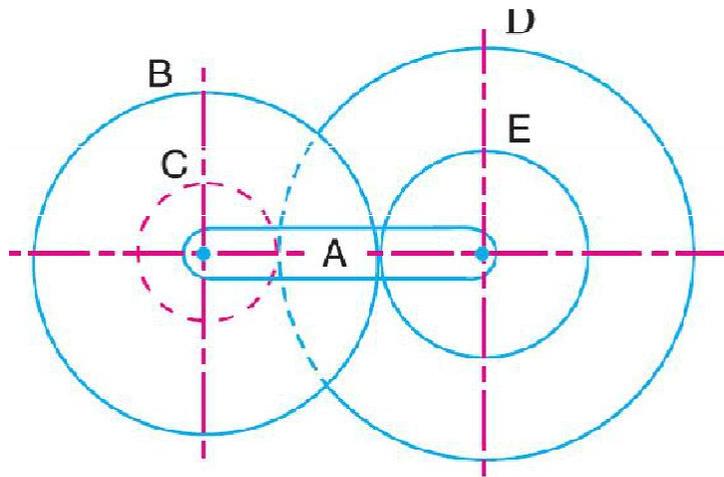
Analysis of Epicyclic Gear trains :

Eg.1



Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (<i>i.e.</i> 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	+ x	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_A}{T_B}$

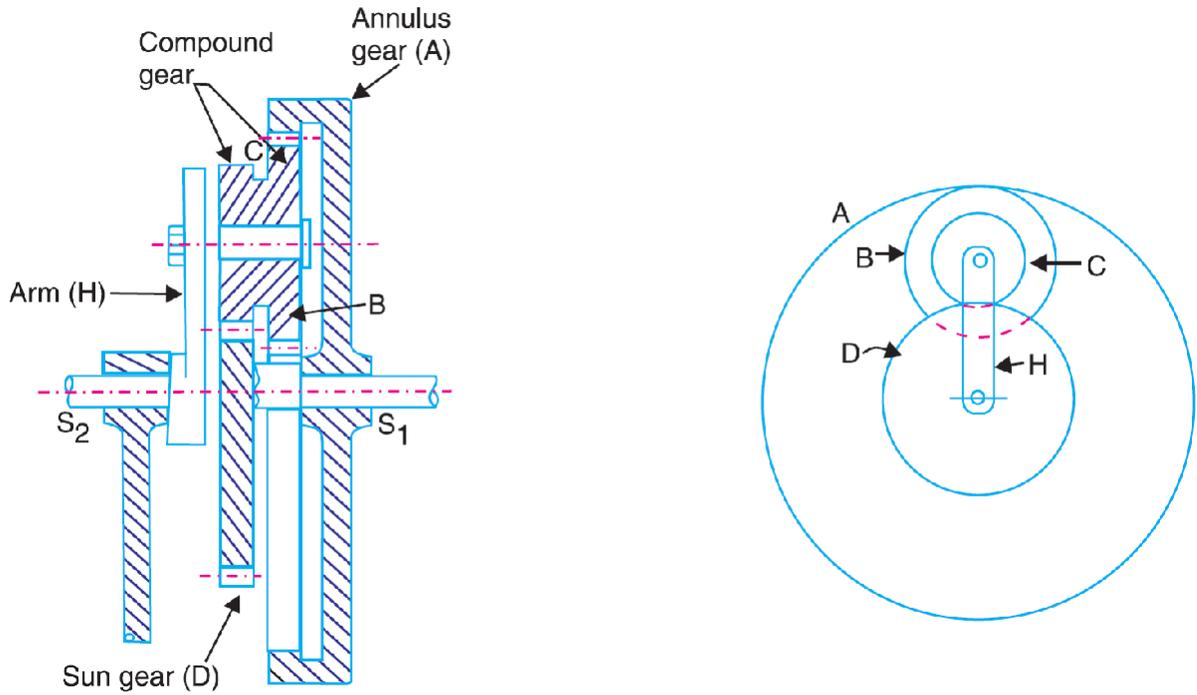
Eg.2



$$T_B + T_E = T_C + T_D$$

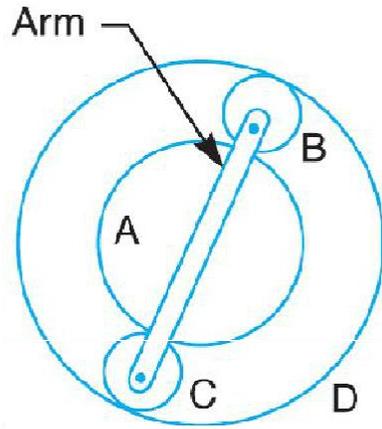
Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear D-E rotated through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear D-E rotated through + x revolutions	0	+x	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + y revolutions to all elements	+y	+y	+y	+y
4.	Total motion	+y	x + y	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Eg.3



Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear D	Compound gear B-C	Gear A
1.	Arm fixed-gear D rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-gear D rotates through + x revolutions	0	+ x	$-x \times \frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_D}{T_C}$	$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$

Eg.4

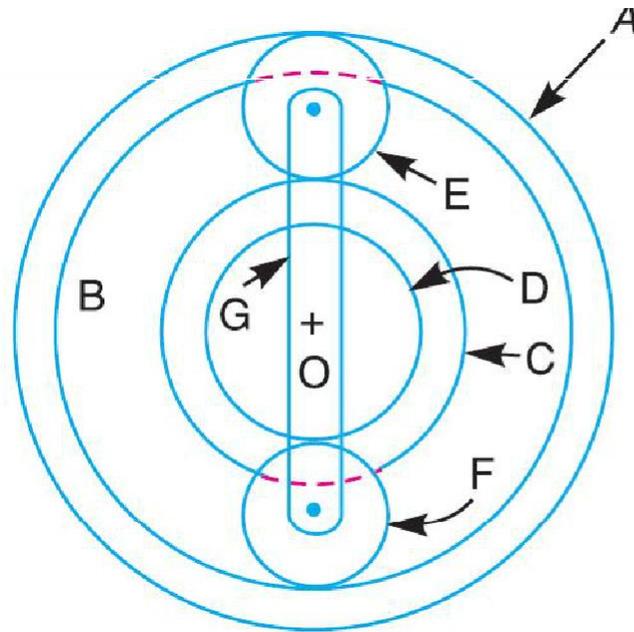


$$d_A + d_B + d_C = d_D$$

$$T_A + 2T_B = T_D$$

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through -1 revolution (<i>i.e.</i> 1 rev. clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B} \times \frac{T_B}{T_D} = +\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x \times \frac{T_A}{T_B}$	$+x \times \frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x - y$	$x \times \frac{T_A}{T_B} - y$	$x \times \frac{T_A}{T_D} - y$

Eg.5

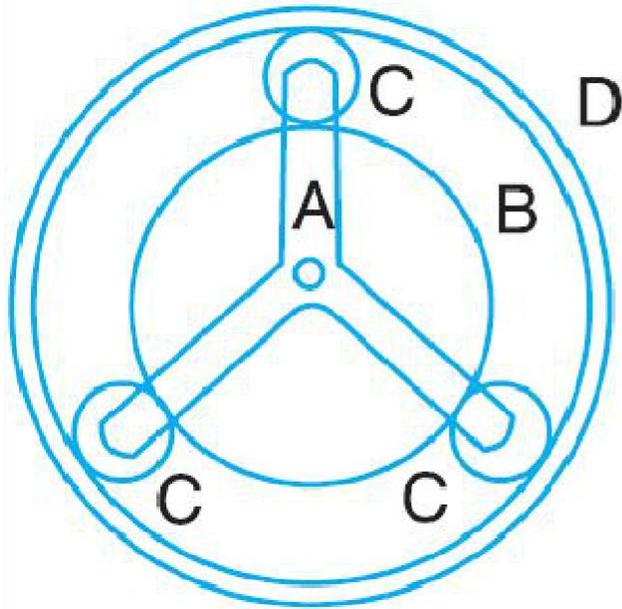


$$T_A = T_C + 2 T_E$$

$$T_B = T_D + 2 T_F$$

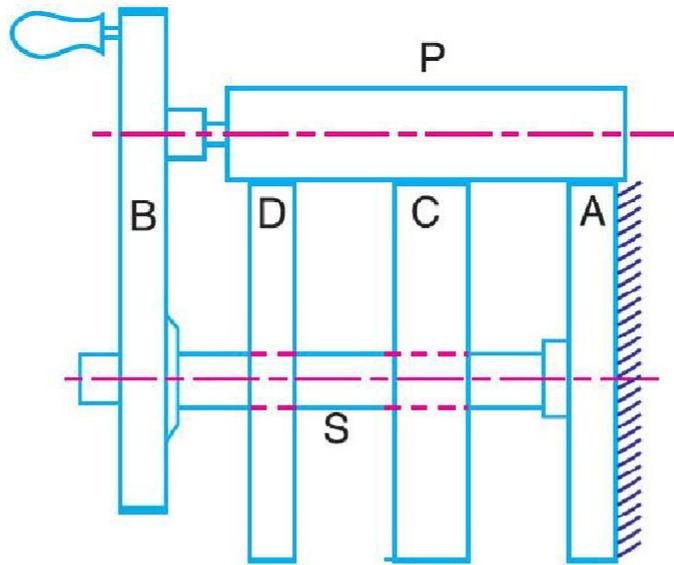
Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel A rotates through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E} \times \frac{T_E}{T_C}$ $= -\frac{T_A}{T_C}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F} \times \frac{T_F}{T_B}$ $= +\frac{T_A}{T_C} \times \frac{T_D}{T_B}$
2.	Arm fixed-wheel A rotates through + x revolutions	0	+x	$+x \times \frac{T_A}{T_E}$	$-x \times \frac{T_A}{T_C}$	$+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$
3.	Add + y revolutions to all elements	+y	+y	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y+x \times \frac{T_A}{T_E}$	$y-x \times \frac{T_A}{T_C}$	$y+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$y+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$

Eg.6



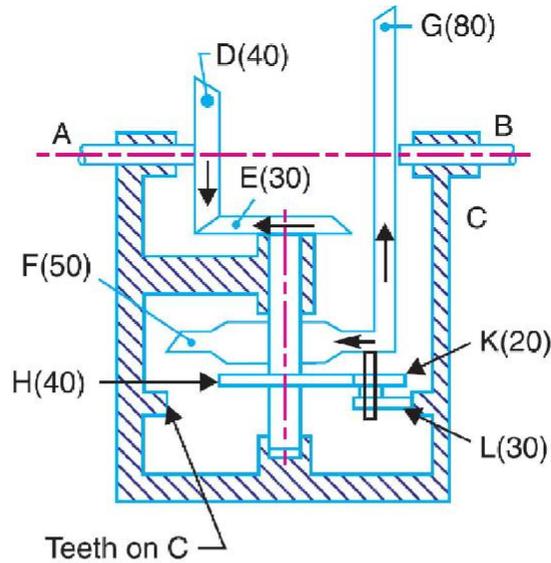
Step No.	Conditions of motion	Revolutions of elements			
		Spider A	Sun wheel B	Planet wheel C	Internal gear D
1.	Spider A fixed, sun wheel B rotates through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_C} \times \frac{T_C}{T_D} = -\frac{T_B}{T_D}$
2.	Spider A fixed, sun wheel B rotates through + x revolutions	0	+ x	$-x \times \frac{T_B}{T_C}$	$-x \times \frac{T_B}{T_D}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_B}{T_C}$	$y - x \times \frac{T_B}{T_D}$

Eg.7



Step No.	Conditions of motion	Revolutions of elements			
		Arm B	Gear A	Gear C	Gear D
1.	Arm B fixed, gear A rotated through + 1 revolution (<i>i.e.</i> 1 revolution anticlockwise)	0	+ 1	$+\frac{T_A}{T_C}$	$+\frac{T_A}{T_C} \times \frac{T_C}{T_D} = +\frac{T_A}{T_D}$
2.	Arm B fixed, gear A rotated through + x revolutions	0	+ x	$+x \times \frac{T_A}{T_C}$	$+x \times \frac{T_A}{T_D}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y + x \times \frac{T_A}{T_C}$	$y + x \times \frac{T_A}{T_D}$

Eg.8

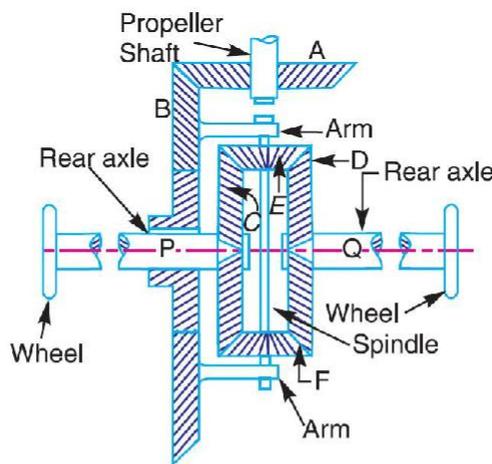


Step No.	Conditions of motion	Revolutions of elements				
		Arm or wheel F	Wheel E	Wheel H	Compound wheel K-L	Wheel C
1.	Arm fixed-wheel E rotated through - 1 revolution (i.e. 1 revolution clockwise)	0	- 1	- 1 (∵ E and H are on the same shaft)	$+\frac{T_H}{T_K}$	$+\frac{T_H}{T_K} \times \frac{T_L}{T_C}$
2.	Arm fixed-wheel E rotated through - x revolutions	0	- x	- x	$+ x \times \frac{T_H}{T_K}$	$+ x \times \frac{T_H}{T_K} \times \frac{T_L}{T_C}$
3.	Add - y revolutions to all elements	- y	- y	- y	- y	- y
4.	Total motion	- y	- x - y	- x - y	$x \times \frac{T_H}{T_K} - y$	$x \times \frac{T_H}{T_K} \times \frac{T_L}{T_C} - y$

Differential of Automobile :

The differential gear used in the rear drive of an automobile is shown in Fig. . Its function is (a) to transmit motion from the engine shaft to the rear driving wheels, and (b) to rotate the rear wheels at different speeds while the automobile is taking a turn. As long as the automobile is running on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than the inner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic gear train with bevel gears as shown in Fig. The bevel gear A (known as pinion) is keyed to the propeller shaft driven from the engine shaft through

universal coupling. This gear *A* drives the gear *B* (known as crown gear) which rotates freely on the axle *P*. Two equal gears *C* and *D* are mounted on two separate parts *P* and *Q* of the rear axles respectively. These gears, in turn, mesh with equal pinions *E* and *F* which can rotate freely on the spindle provided on the arm attached to gear *B*. When the automobile runs on a straight path, the gears *C* and *D* must rotate together. These gears are rotated through the spindle on the gear *B*. The gears *E* and *F* do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears *D* and *C*, the gears *E* and *F* start rotating about the spindle axis and at the same time revolve about the axle axis. Due to this epicyclic effect, the speed of the inner rear wheel decreases by a certain amount and the speed of the outer rear wheel increases, by the same amount



Step No.	Conditions of motion	Revolutions of elements			
		Gear B	Gear C	Gear E	Gear D
1.	Gear <i>B</i> fixed-Gear <i>C</i> rotated through + 1 revolution (<i>i.e.</i> 1 revolution anticlockwise)	0	+ 1	$+\frac{T_C}{T_E}$	$-\frac{T_C}{T_E} \times \frac{T_E}{T_D} = -1$ ($\because T_C = T_D$)
2.	Gear <i>B</i> fixed-Gear <i>C</i> rotated through + <i>x</i> revolutions	0	+ <i>x</i>	$+x \times \frac{T_C}{T_E}$	- <i>x</i>
3.	Add + <i>y</i> revolutions to all elements	+ <i>y</i>	+ <i>y</i>	+ <i>y</i>	+ <i>y</i>
4.	Total motion	+ <i>y</i>	<i>x</i> + <i>y</i>	$y + x \times \frac{T_C}{T_E}$	<i>y</i> - <i>x</i>

From the table, we see that when the gear *B*, which derives motion from the engine shaft, rotates at *y* revolutions, then the speed of inner gear *D* (or the rear axle *Q*) is less than *y* by *x* revolutions and the speed of the outer gear *C* (or the rear axle *P*) is greater than *y* by *x* revolutions. In other words, the two parts of the rear axle and thus the two wheels rotate at two different speeds

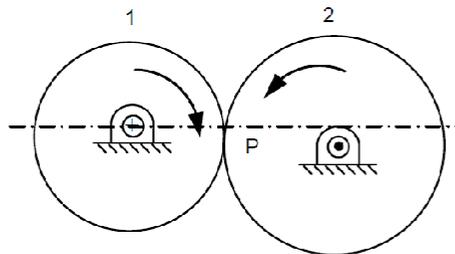
UNIT V
Part-B
GEAR TRAINS

A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. They often consist of multiple gears in the train.

Types of Gear trains :

1. Simple Gear Train

In this gear train, there are series of gears which are capable of receiving and transmitting motion from one gear to another. They may mesh externally or internally. Each gear rotates about separate axis fixed to the frame. Figure shows two gears in external meshing. Let t_1, t_2 be number of teeth on gears 1 and 2.

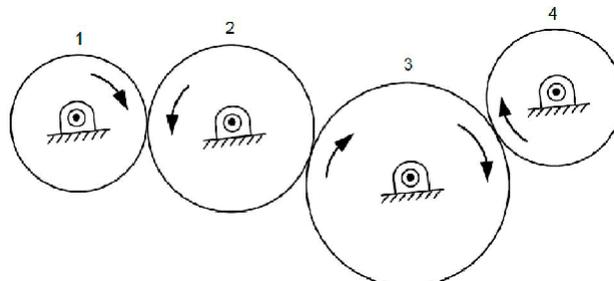


Let N_1, N_2 be speed in rpm for gears 1 and 2. The velocity of P ,

$$V_P = \frac{2\pi N_1 d_1}{60} = \frac{2\pi N_2 d_2}{60}$$

$$\therefore \frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{t_2}{t_1}$$

The two meshing gears in external meshing rotate in opposite sense whereas in internal meshing they rotate in same sense. In simple gear train, there can be more than two gears also as shown in Figure



Let N_1, N_2, N_3, \dots be speed in rpm of gears 1, 2, 3, \dots etc., and t_1, t_2, t_3, \dots be number of teeth of respective gears 1, 2, 3, \dots , etc.

In this gear train, gear 1 is input gear, gear 4 is output gear and gears 2, 3 are intermediate gears.

The gear ratio of the gear train is give by

$$\text{Gear Ratio} = \frac{N_1}{N_4} = \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4}$$

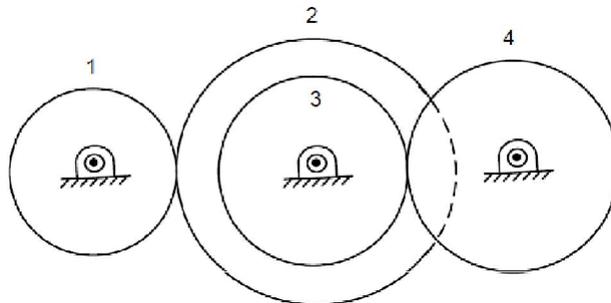
$$\therefore \frac{N_1}{N_2} = \frac{t_2}{t_1}; \quad \frac{N_2}{N_3} = \frac{t_3}{t_2} \quad \text{and} \quad \frac{N_3}{N_4} = \frac{t_4}{t_3}$$

$$\text{Therefore,} \quad \frac{N_1}{N_4} = \frac{t_2}{t_1} \times \frac{t_3}{t_2} \times \frac{t_4}{t_3} = \frac{t_4}{t_1}$$

This expression indicates that the intermediate gears have no effect on gear ratio. These intermediate gears fill the space between input and output gears and have effect on the sense of rotation of output gear

2. Compound Gear Train

In this type of gear train, at least two gears are mounted on the same shaft and they rotate at the same speed. This gear train is shown in Figure 3.30 where gears 2 and 3 are mounted on same shaft and they rotate at the same speed, i.e. N_2, N_3



Let N_1, N_2, N_3, \dots be speed in rpm of gears 1, 2, 3, \dots , etc. and t_1, t_2, t_3, \dots , etc. be number of teeth of respective gears 1, 2, 3, \dots , etc.

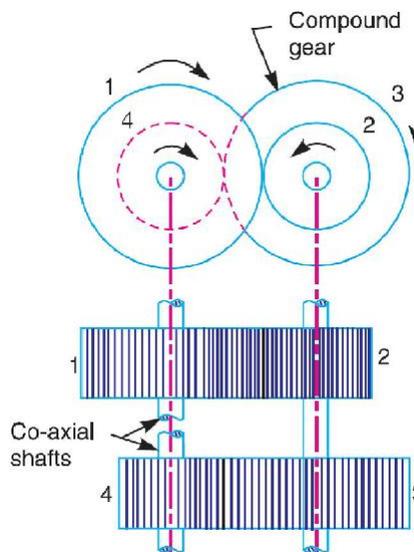
$$\begin{aligned} \text{Gear Ratio} &= \frac{N_1}{N_4} = \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4} \\ &= \frac{t_2}{t_1} \times \frac{t_4}{t_3} \end{aligned}$$

Therefore, unlike simple gear train the gear ratio is contributed by all the gears. This gear train is used in conventional automobile gear box.

3. Reverted Gear Train:

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train as shown in Fig. We see that gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is like

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$



Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same

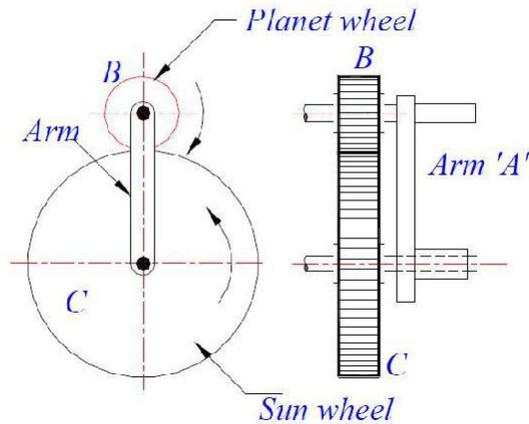
$$r_1 + r_2 = r_3 + r_4$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius

$$T_1 + T_2 = T_3 + T_4$$

4. Epicyclic gear train:

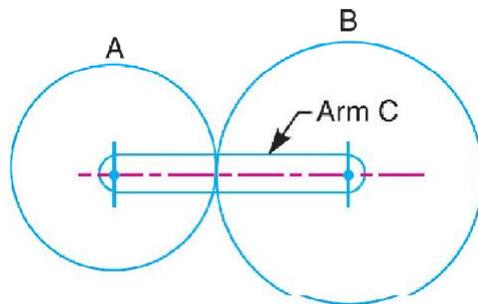
Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun. Here is a picture of a typical gear box. This design can produce large gear ratios in a small space and are used on a wide range of applications from marine gearboxes to electric screwdrivers.



The diagram shows a gear B on the end of an arm. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.

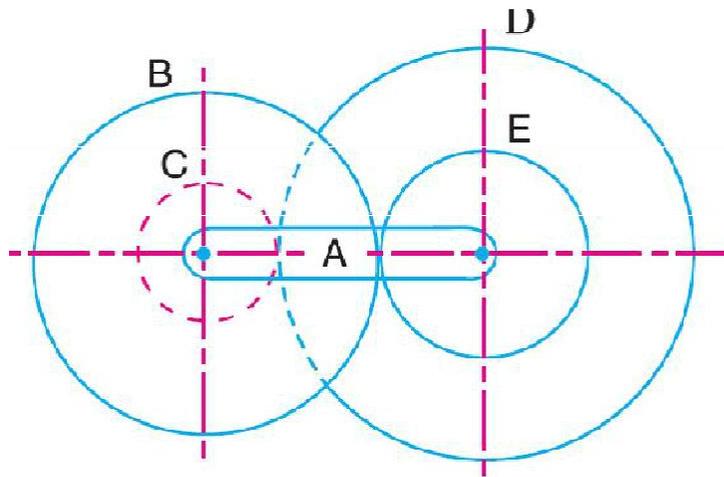
Analysis of Epicyclic Gear trains :

Eg.1



Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (<i>i.e.</i> 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	+ x	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_A}{T_B}$

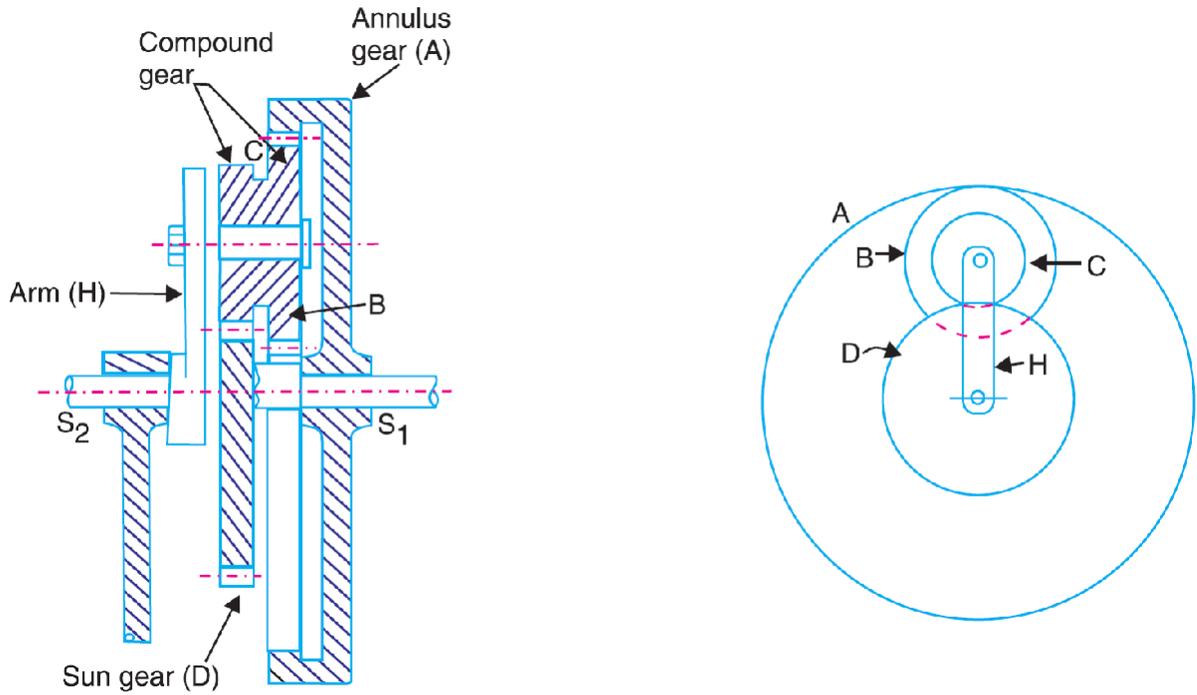
Eg.2



$$T_B + T_E = T_C + T_D$$

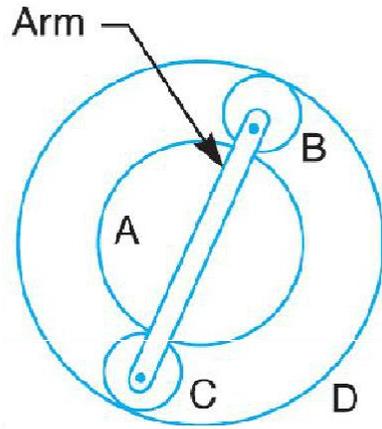
Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear D-E rotated through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear D-E rotated through + x revolutions	0	+x	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + y revolutions to all elements	+y	+y	+y	+y
4.	Total motion	+y	x + y	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Eg.3



Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear D	Compound gear B-C	Gear A
1.	Arm fixed-gear D rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-gear D rotates through + x revolutions	0	+ x	$-x \times \frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_D}{T_C}$	$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$

Eg.4

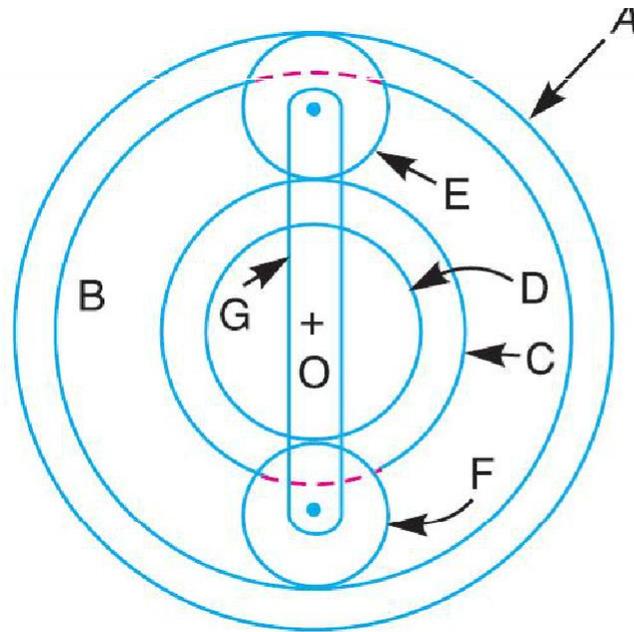


$$d_A + d_B + d_C = d_D$$

$$T_A + 2T_B = T_D$$

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through -1 revolution (<i>i.e.</i> 1 rev. clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B} \times \frac{T_B}{T_D} = +\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x \times \frac{T_A}{T_B}$	$+x \times \frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x - y$	$x \times \frac{T_A}{T_B} - y$	$x \times \frac{T_A}{T_D} - y$

Eg.5

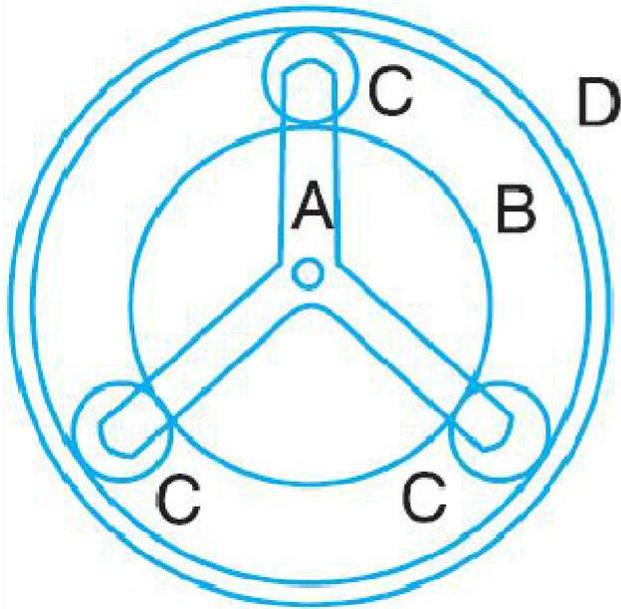


$$T_A = T_C + 2 T_E$$

$$T_B = T_D + 2 T_F$$

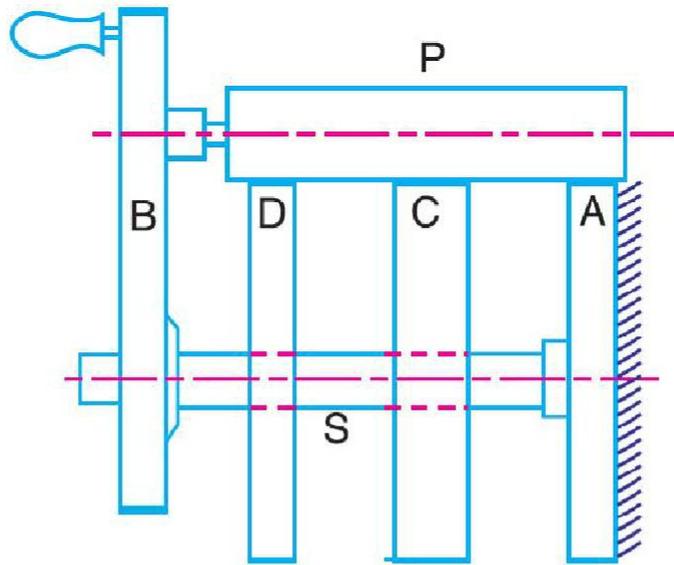
Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel A rotates through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E} \times \frac{T_E}{T_C}$ $= -\frac{T_A}{T_C}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F} \times \frac{T_F}{T_B}$ $= +\frac{T_A}{T_C} \times \frac{T_D}{T_B}$
2.	Arm fixed-wheel A rotates through + x revolutions	0	+x	$+x \times \frac{T_A}{T_E}$	$-x \times \frac{T_A}{T_C}$	$+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$
3.	Add + y revolutions to all elements	+y	+y	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y+x \times \frac{T_A}{T_E}$	$y-x \times \frac{T_A}{T_C}$	$y+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$y+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$

Eg.6



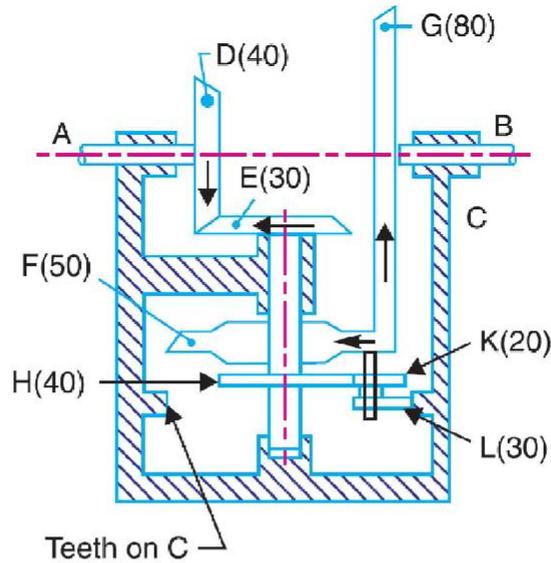
Step No.	Conditions of motion	Revolutions of elements			
		Spider A	Sun wheel B	Planet wheel C	Internal gear D
1.	Spider A fixed, sun wheel B rotates through + 1 revolution (i.e. 1 rev. anticlockwise)	0	+ 1	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_C} \times \frac{T_C}{T_D} = -\frac{T_B}{T_D}$
2.	Spider A fixed, sun wheel B rotates through + x revolutions	0	+ x	$-x \times \frac{T_B}{T_C}$	$-x \times \frac{T_B}{T_D}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_B}{T_C}$	$y - x \times \frac{T_B}{T_D}$

Eg.7



Step No.	Conditions of motion	Revolutions of elements			
		Arm B	Gear A	Gear C	Gear D
1.	Arm B fixed, gear A rotated through + 1 revolution (<i>i.e.</i> 1 revolution anticlockwise)	0	+ 1	$+\frac{T_A}{T_C}$	$+\frac{T_A}{T_C} \times \frac{T_C}{T_D} = +\frac{T_A}{T_D}$
2.	Arm B fixed, gear A rotated through + x revolutions	0	+ x	$+x \times \frac{T_A}{T_C}$	$+x \times \frac{T_A}{T_D}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y + x \times \frac{T_A}{T_C}$	$y + x \times \frac{T_A}{T_D}$

Eg.8

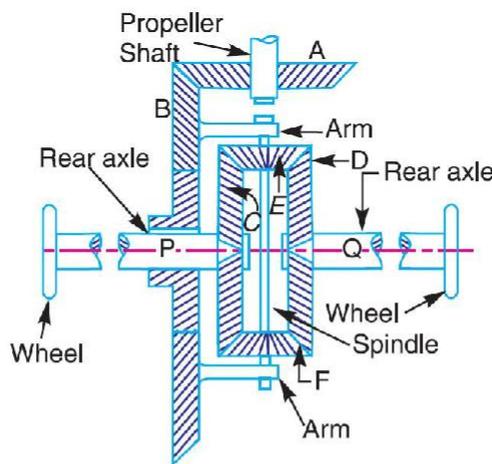


Step No.	Conditions of motion	Revolutions of elements				
		Arm or wheel F	Wheel E	Wheel H	Compound wheel K-L	Wheel C
1.	Arm fixed-wheel E rotated through - 1 revolution (i.e. 1 revolution clockwise)	0	- 1	- 1 (∵ E and H are on the same shaft)	$+\frac{T_H}{T_K}$	$+\frac{T_H}{T_K} \times \frac{T_L}{T_C}$
2.	Arm fixed-wheel E rotated through - x revolutions	0	- x	- x	$+ x \times \frac{T_H}{T_K}$	$+ x \times \frac{T_H}{T_K} \times \frac{T_L}{T_C}$
3.	Add - y revolutions to all elements	- y	- y	- y	- y	- y
4.	Total motion	- y	- x - y	- x - y	$x \times \frac{T_H}{T_K} - y$	$x \times \frac{T_H}{T_K} \times \frac{T_L}{T_C} - y$

Differential of Automobile :

The differential gear used in the rear drive of an automobile is shown in Fig. . Its function is (a) to transmit motion from the engine shaft to the rear driving wheels, and (b) to rotate the rear wheels at different speeds while the automobile is taking a turn. As long as the automobile is running on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than the inner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic gear train with bevel gears as shown in Fig. The bevel gear A (known as pinion) is keyed to the propeller shaft driven from the engine shaft through

universal coupling. This gear *A* drives the gear *B* (known as crown gear) which rotates freely on the axle *P*. Two equal gears *C* and *D* are mounted on two separate parts *P* and *Q* of the rear axles respectively. These gears, in turn, mesh with equal pinions *E* and *F* which can rotate freely on the spindle provided on the arm attached to gear *B*. When the automobile runs on a straight path, the gears *C* and *D* must rotate together. These gears are rotated through the spindle on the gear *B*. The gears *E* and *F* do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears *D* and *C*, the gears *E* and *F* start rotating about the spindle axis and at the same time revolve about the axle axis. Due to this epicyclic effect, the speed of the inner rear wheel decreases by a certain amount and the speed of the outer rear wheel increases, by the same amount



Step No.	Conditions of motion	Revolutions of elements			
		Gear B	Gear C	Gear E	Gear D
1.	Gear <i>B</i> fixed-Gear <i>C</i> rotated through + 1 revolution (<i>i.e.</i> 1 revolution anticlockwise)	0	+ 1	$+\frac{T_C}{T_E}$	$-\frac{T_C}{T_E} \times \frac{T_E}{T_D} = -1$ ($\because T_C = T_D$)
2.	Gear <i>B</i> fixed-Gear <i>C</i> rotated through + <i>x</i> revolutions	0	+ <i>x</i>	$+x \times \frac{T_C}{T_E}$	- <i>x</i>
3.	Add + <i>y</i> revolutions to all elements	+ <i>y</i>	+ <i>y</i>	+ <i>y</i>	+ <i>y</i>
4.	Total motion	+ <i>y</i>	<i>x</i> + <i>y</i>	$y + x \times \frac{T_C}{T_E}$	<i>y</i> - <i>x</i>

From the table, we see that when the gear *B*, which derives motion from the engine shaft, rotates at *y* revolutions, then the speed of inner gear *D* (or the rear axle *Q*) is less than *y* by *x* revolutions and the speed of the outer gear *C* (or the rear axle *P*) is greater than *y* by *x* revolutions. In other words, the two parts of the rear axle and thus the two wheels rotate at two different speeds